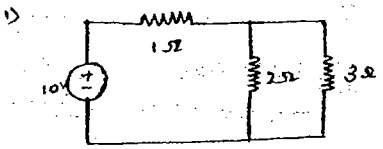
Electrical Energy

importance - inter convertable

- transportation

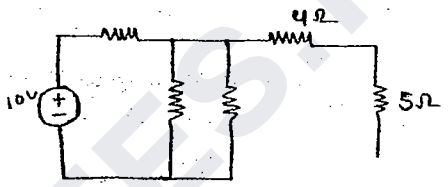
Circuit



The arrangement of components in interconnected form to act together where current i is intended (⊙) necessary to flow through all the elements.

(So closed path concept in ckt)

Network

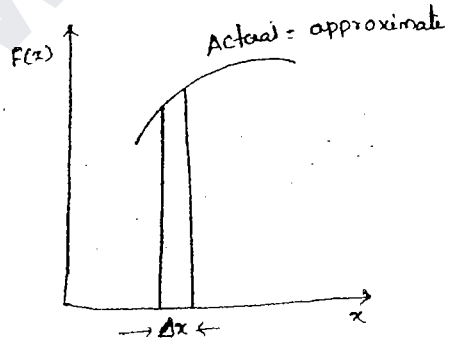


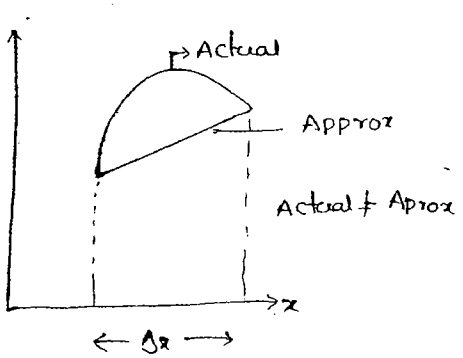
→ 4Ω and 5Ω branch is floating current will not flow through them. Hence it is Network

→ our electrical power system is very big N/w but we do Circuit analysis (per unit Symmetrical faults, Stability .. etc)

→ All practical electrical components such as motors, heaters etc are modeled as a circuit in terms of basic ckt components ie R, L, C, V, I_dc

Linearity & Non Linearity





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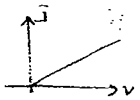
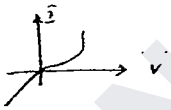
→ Most of the practical system in nature are non linear but any non-linear system can be made to operate in linear mode by considering small incremental changes in time but the same system for sudden big changes undergo non linear mode of operation.

* All the electrical components of ckt's of N/w which obey Ohm's law homogenity & Superposition principal are linear

Resistor – linear 

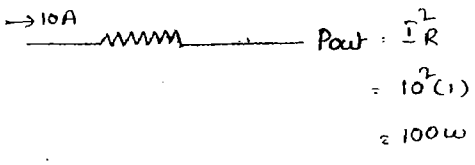
diode – Nonlinear 

By observing the VI characteristics of any component we can identify whether it is linear or not

Bilateral and unilateral

→ property
→ Behaviour
→ characteristics

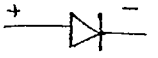
are independent to current direction & voltage polarity they are bilateral

Eg: 

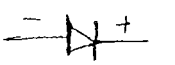
unilateral

→ property
→ Behaviour
→ characteristics

are dependent upon the 'i' direction 'v' polarity it is unilateral.



FB → conduct
↓ R = 0



RB R = ∞

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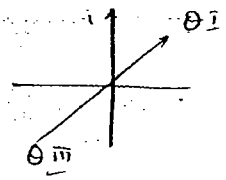
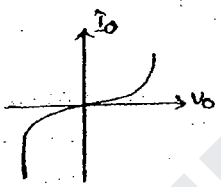
→ our Electrical power Tr/lines & Telephone lines are Bilateral

→ we can identify the bilateral principle if the V-I characteristics are Symmetrical in 1st & IIIrd Quadrant

Bilateral



Unilateral



Passive & active Elements

Sources / Active Element

→ give out
→ drive
→ deliver
} E.E

p deliver is +ve

p absorbed is −ve

Sink / passive Element:

→ absorb → convert → disipate
→ store → waste

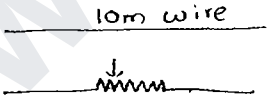
p absorbed +ve

p delivered −ve

Resistor, L, C

Lumped parameters:- Ex: Short Tr/lines

10m wire

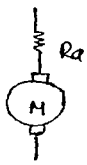


→ having Resistance of 10Ω

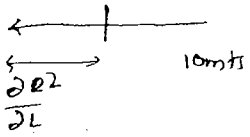
→ Such Representation is Called Lumped representation

Eqn's are linear algebric Eqn's

Eg:



Distributed parameters:-



Eg: Long Tr/lines

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→ Equation's are differential

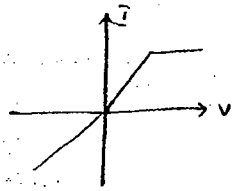
→ Relation line, $v = A\lambda$

→ Relation like this can only varied with distributed parameters

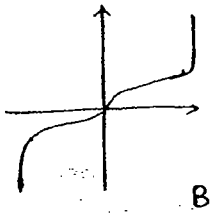
Q. The V-I characteristics of certain elements are shown identify.

a) Linearity b) Biletral / uniletral c) active / passive

①

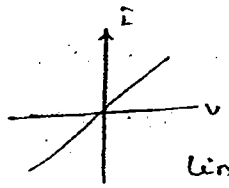


Non linear, uniletral

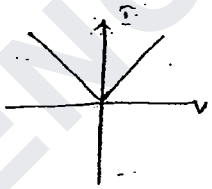


Biletral non linear

③



Linear, Biletral.



linear, uniletral
Active (or II)

Node A node is a point of inter connection or function b/w two or more components.

Branch: A branch is an elemental connection b/w two nodes, every element by it self is a brach

Mesh: A Mesh is a closed path in a ckt or N/w which should not have furthure closed path in it

Loop: A loop is a closed path of a ckt or N/w which may have closed paths in it

NOTE: 1) for any ckt meshes No of meshes: $(m) = b - n + 1$

The no of min^m equations to store a n/w or a ckt are equal to mesh equations

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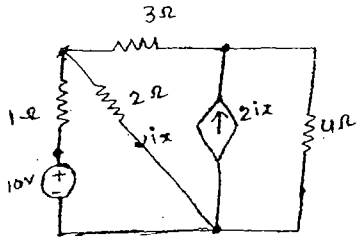
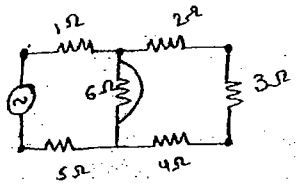
① 

nodes - 4

branches - 6

meshes = 3 (b-n+1)

= 6-4+1 = 3

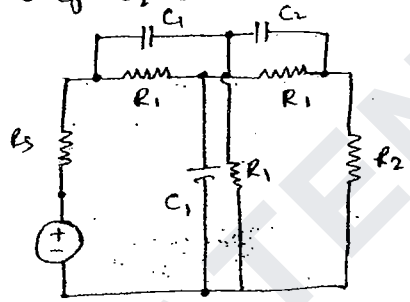
② 

Nodes - 5

Branches - 7

m - 7-5+1 = 3

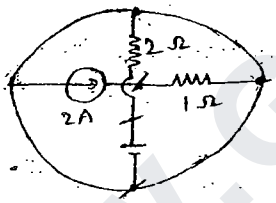
③ The Min^m No of eqn's to solve the N/w Below



nodes - 6

branch - 9

meshes - 9-6+1 = 4

④ 

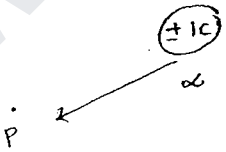
No of Min^m eqn's to solve N/w

Nodes - 3

Branches - 4

m - 4-3+1 = 2

Voltage: The Force which drives charges.



$V = \frac{dw}{dq}$

Symbols v, V, $\bar{V}$ (V: Average; $\bar{V}$: AC)

$1V = \frac{1J}{1C}$

unit - volts

Voltage → Source

AC DC

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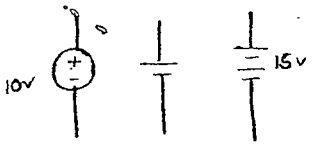
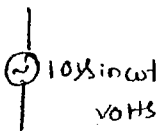
DC



Ex: Cell, Battery, Fuel cel
Dc generator

AC

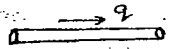


Ex: Alternators
Inverter
ups.

mV, V, kV, MV

Current:-

Rate of Flow of charge



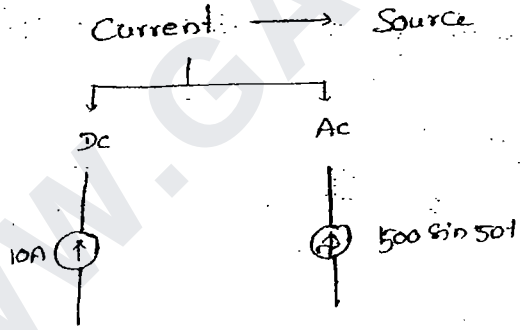
$$i = \frac{dq}{dt} = \frac{Q}{t}$$

units - Amperes

$$1A = \frac{1C}{1sec}$$

1C of charge taken in 1sec

Range: μA, mA, A, kA



practically there is no Current Source

DC Series generator can be modeled as DC Current Source
AC Series generator Can be " " Ac Current Source
Feeders

Analysis i/p → (Ckt) → o/p

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Resistance:-

Resistance is a Electrical property of matter

Resistor is a Component

→ It is a passive Element as it dissipates Electrical Energy in the form of Heat

units ohm (Ω)

Range: $\mu\Omega$, $m\Omega$, Ω, $K\Omega$, $M\Omega$

→ Resistance in Tr/line ⟶ Lumped in short
distributed in Long

$V = IR$

$I = V/R$

$$R = \frac{\rho l}{A}$$

ρ - Resistivity of material ↑ depends upon Nature of Material does not Temp, Length ...

l → Length of the Material

a → Cross Sectional Area of the material.

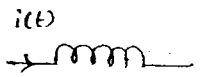
$$\rho = \frac{R.a}{l}$$

units for Resistivity : Ω-m

Inductance:-

it is a Electro magnetic (Ferro) material

Inductor is a Component



Coiled wire providing with time Varying Current

$i(t)$ — time Varying

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→ It is a passive element since it stores electrical energy in its electromagnetic form.

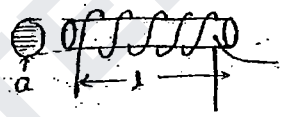
units Henry (H)

Range: μH, mH^*, H

Example:- choke coil etc

$$\boxed{V = L\frac{di}{dt}} \quad \& \quad \boxed{i = \frac{1}{L}\int v\,dt}$$

Constructional/ Formulae design – $\boxed{L = \frac{\mu N^2 a}{l}}$ Henry



μ - permeability of Material

$\mu = \mu_0 \mu_r$. $\mu_0 = 4\pi \times 10^{-7}$

$\mu_r = 1$ (air)

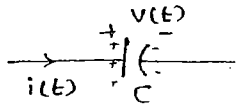
N - No of turns of coil

a - Crossectional area of the winding

l - length in the flux part Material.

Capacitance:-

Capacitor is a Component



→ It is a passive element since it stores electrical energy in its electro static form.

units – Farads

Range – PF, nF, μF^*, mF

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Ex: power factor, Correcting Equipment

$$i = C\frac{dv}{dt} \Rightarrow v = \frac{1}{C}\int i\,dt$$

$$C = \frac{\epsilon A}{d}$$

A - area of cross sectio

ϵ - permitibity of dielectric

d - distance b/w plates

$\epsilon = \epsilon_0 \epsilon_r$

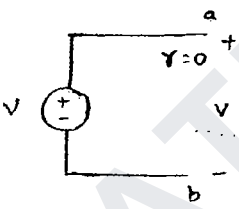
$\epsilon_0 = 8.85 \times 10^{-12}$ F/m

$\epsilon_r = 1$ (air)

$R_t = R_0(1+\alpha t)$

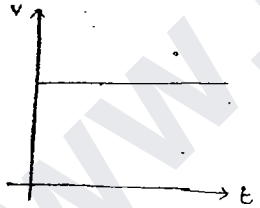
Ideal voltage Source:-

Internal Resistance - 0

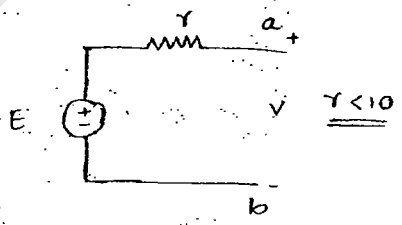


∴ Replaced by Short ckt

in terms of Resistance



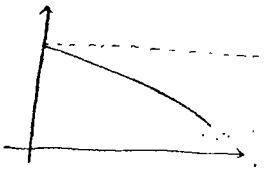
practical voltage Source



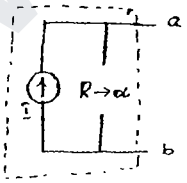
E - internal EMF

if E > V - discharging [source]

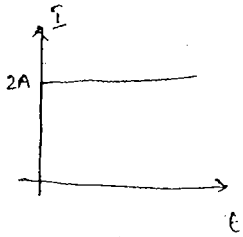
E < V - charging [sink]



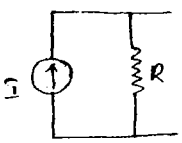
Ideal Current Source:-



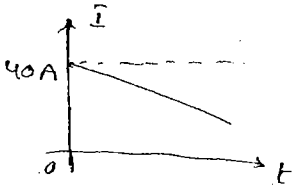
R → ∞



practical Current Source



R → MΩ



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→ DC System / Time Invariant Systems.

DC Suppliers are

→ unipolar

→ unidirectional

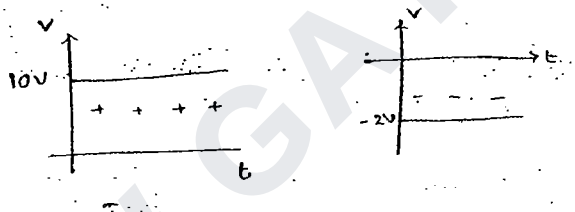
→ No change in phase / polarity

→ power frequency = 0 Hz

Ex: They are used in Small, Independent Isolated Systems, where Electrical Energy Can be Stored in Small Quantities

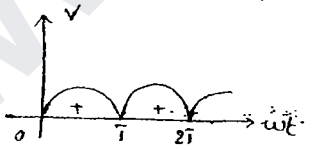
Ex: Cell phones, Machine tools, Defence System etc.

→ Standard DC voltage wave form



$T \to \infty$

$F \to 0$



$T = \frac{1}{f}$ → Ripple frequency

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Ac Systems / Time variant System :-

Ac Suppliers are
- Bipolar
- Bi directional
- Definite change in phase / polarity
- power frequency Exist

India - 50 HZ

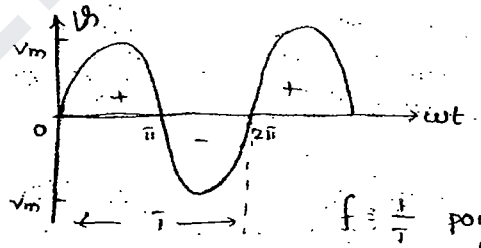
U.S.A - 60 HZ

They are used in Bulk powered Electrical System where Energy Stored is Not possible

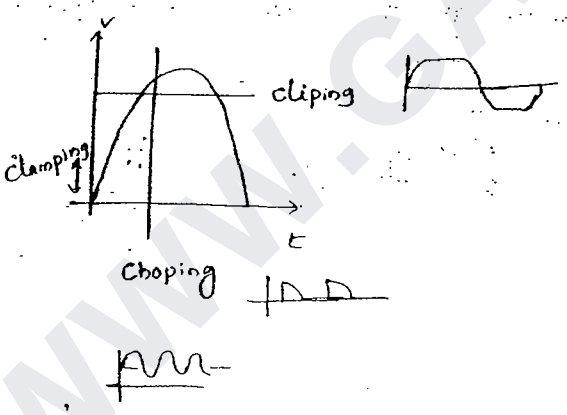
Ex: Generation, Transmission, distribution, utilization } A.C

stored Ac voltage wave form:

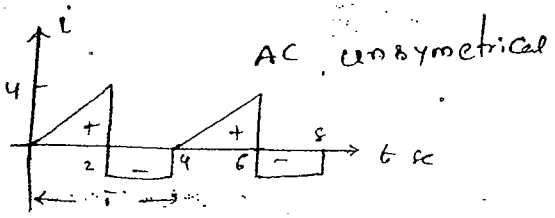
$v = V_m \sin \omega t$



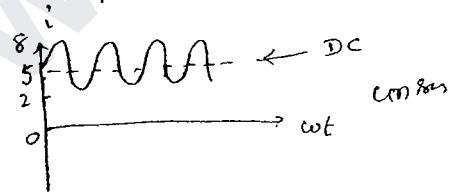
$f = \frac{1}{T}$ power frequency.



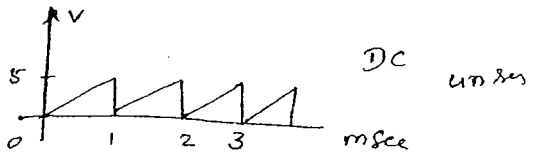
(P)



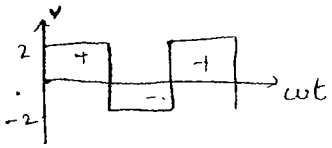
AC unsymetrical



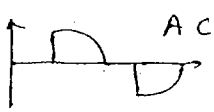
unsym

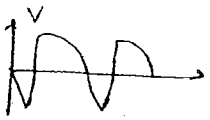


DC unsym



AC Symetrical





Avg voltage is +ve DC

it has Ac Components

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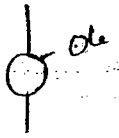
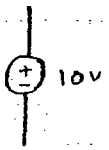
→ Independent Source:-

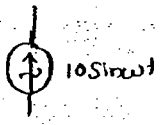
- Values
- Behaviour
- properities

} is independent to any other External parameter, with in (or) out Side the ckt.



Ex:





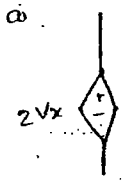
dependent Sources:-

- Values
- magnitude
- Behaviour
- properity

} depends Some External parameters within (or) out Side the Ckt.

→ Four types.

(a)



Voltage dependent Voltage Source

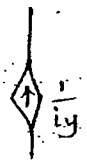
VCVS

(b)

CDVS

CCVS

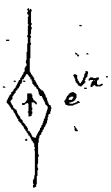
(c)



CDCS

CCCS

(d)



VDCS

VCCS

dependent Sources have a finite value of Resistance/impedance So we can't Suppress them.

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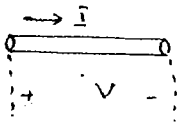
ohm's law :-

constant temperature & uniform Cross Section of Conductor

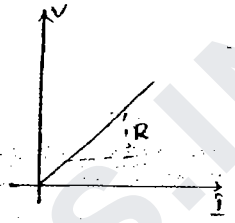
$V \propto i$



$V = IR$

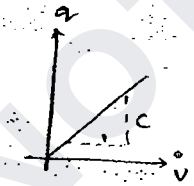
↳ constant proportionality

Resistance : Ω



ohm's law for Capacitor :-

$q = CV$



$$\frac{dq}{dt} = C\frac{dv}{dt} + V\frac{dc}{dt}$$

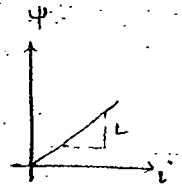
$$\frac{dq}{dt} = C\frac{dv}{dt} \Rightarrow i = C\frac{dv}{dt}$$

ohm's law for Inductor :-

$\psi = LI$ — ψ - flux linkages

$= N\phi$

$LI = N\phi$



$$N\frac{d\phi}{dt} = L\frac{di}{dt} \Rightarrow V = L\frac{di}{dt}$$

power (P) :-

$$P = \frac{dE}{dt} = \frac{dw}{dt}$$

units - watts (w)

1 hp = 746 watts

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R

$P_R = V_R i_R = i_R^2 R = \frac{V_R^2}{R}$

1 kwh = 1000w x 1h

= 500w x 2

= 100w x 10hrs

= 2000w x $\frac{1}{2}$

L

A pure Inductor never dissipates Energy

But Instantaneous power Exist

$P_L = Li\frac{di}{dt}$ watts.

↑

Instantaneous

C

A pure Capacitor Never dissipate Energy

But instantaneous power Exist

$P_C = CV\frac{dv}{dt}$ watts.

↓

Instantaneous.

Energy:

Energy is the Capacity to do work

$E = \int p\,dt$

units: General unit - joules

Heat - Calories

Electrical Energy unit - kwH

② A 40 w incandescent bulb turned on for — No of days will produce 1 unit of Electrical Energy.

$\frac{25}{40}$

1 unit - 1 kwh

40 w x $\frac{25}{\uparrow}$ h = 1000 w

25 hours - 1 day

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R

Energy $E_R = i^2 R t = \frac{v^2}{R} t = v \cdot i \cdot t$ joules

L

Energy $E_L = \int p\,dt$

$$E_L = \int Li \frac{di}{dt} dt = \frac{1}{2} Li^2 = \frac{1}{2} \psi i = \frac{\psi^2}{2L^2}$$

$\psi = LI$

C

Energy $E_C = \int Cv \frac{dv}{dt} dt$

$$= \int Cv \cdot dv = \frac{1}{2} Cv^2 = \frac{1}{2} qv = \frac{q^2}{2C} \text{ joules}$$

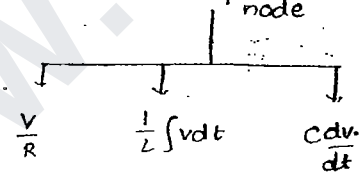
1 kwH = 36×10^5 joules.

→ power at an instant — Energy over a period.

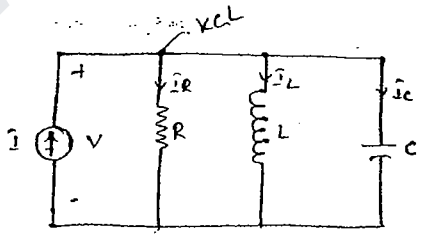
Kirchoff's Law:-

→ Kirchoff's 1st Law | Kirchoff Current law | Kirchoff Nodal law

$\sum i\big|_{node} = 0$ algebric Sum of Current at a node '0'



Ex:

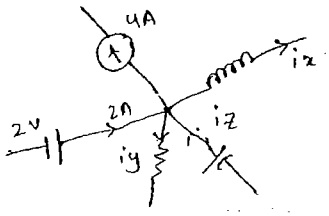


$I = i_R - i_L - i_C = 0$

$I = i_R + i_L + i_C$

$I = \frac{V}{R} + \frac{1}{L} \int v\,dt + C \frac{dv}{dt}$

(2)



KCL :

$+2 - 4 - ix + iz - iy = 0$

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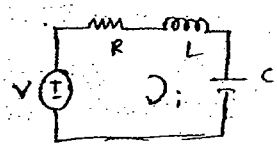
Kirchoff 2nd law / K.V.L / K. mesh law :-

$$\sum V\Big|_{mesh} = 0$$

iR , $L\frac{di}{dt}$, $\frac{1}{C}\int i\,dt$

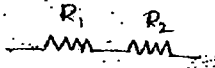
Ex:



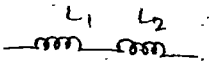
$$-V + iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0$$

Series Connection of elements :

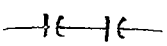
When i through the elements Equal.



$R_1 + R_2$

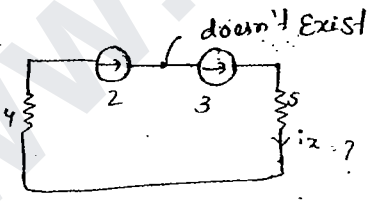


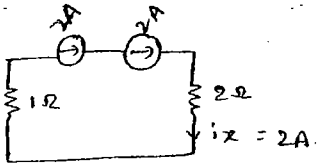
$L_1 + L_2$

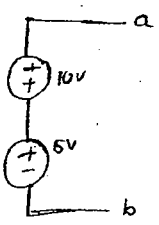


$\frac{C_1 C_2}{C_1 + C_2}$

Ex:







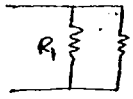
Two Current Sources of different values Can never Exist in Series. However if they Exists there values must be Equal.

→ There is no Restriction in Connecting voltage Sources in Series

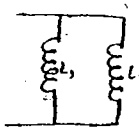
WWW.GATENOTES.IN

Parallel connection:

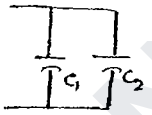
When voltage across them are Equal



$\Rightarrow \frac{R_1 R_2}{R_1 + R_2}$

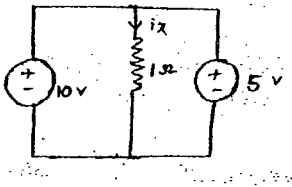


$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2}$

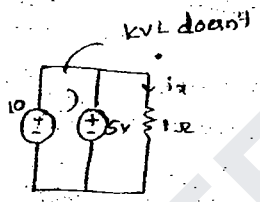


$C_p = C_1 + C_2$

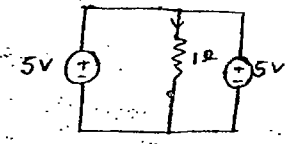
Ex:



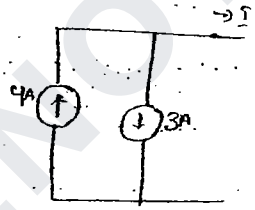
$\Rightarrow$



$-10 + 5 \neq 0$



$i_x = 5V$



$I = 1A$

→ Never two ideal voltage sources of different values can existing parallel. However if they exist they must be Equal.

→ There is No Restriction in connecting Current Sources in parallel.

Open Ckt: (OC):

→ In an open ckt $i = 0$ for any 'v'

$$R_{O.C} = \frac{V}{i} = \frac{\infty}{=}$$

O.C in Series with the element. the element can be neglected.

Short Ckt: (S.C):

In a short ckt, $V = 0$ for any i Current

$$R_{SC} = \frac{0}{i} = 0\Omega$$

Any element in parallel to S.C, that element can be neglected.

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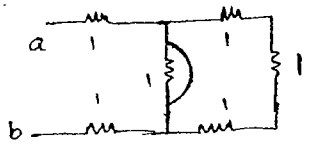
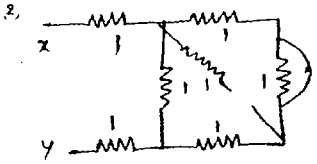
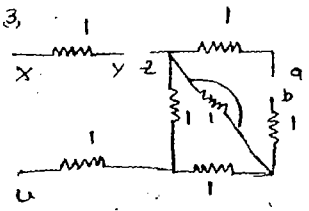
(P)



$R_{ab} = 2\Omega$



$R_{xy} = 8/3$



$R_{xy} = 1$

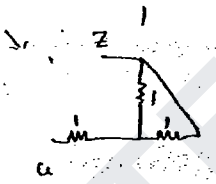
$R_{yz} - \infty$

$R_{xz} = \infty$ / o.c

$R_{yu} - \infty$

$R_{xu} = \infty$ / o.c

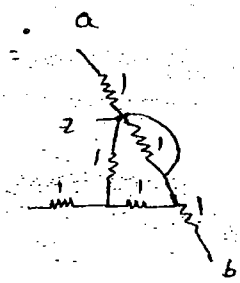
$R_{zu} -$



$(1 \| 1) + 1$

R_{zu} $3/2$

$R_{ab} =$



$= 2\Omega$

Conductance:-

Ability to conduct

use to classify Conductance.

Best Conductor → Silver } Economical
→ Gold }
→ Copper → when Current densities
→ Al large compact systems
↲ like M/c, T/F etc

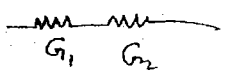
used where Low Cost

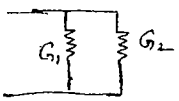
weight is less Ex: out electrical T&D systems

units - Mho (Ω), Siemens (S)

$V = iR = \frac{i}{G}$ & $i = VG$

$G = \frac{1}{R}$





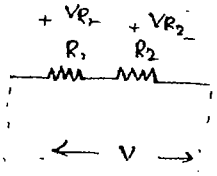
$G_p = G_1 + G_2$

$\frac{1}{G_s} = \frac{1}{G_1} + \frac{1}{G_2}$

WWW.GATENOTES.IN

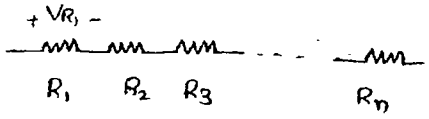
Voltage division Rule:-

↳ For Series Connected elements only.



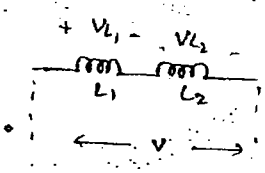
$$V_{R_1} = V\left[\frac{R_1}{R_1+R_2}\right]$$

$$V_{R_2} = V\left[\frac{R_2}{R_1+R_2}\right]$$



$$V_{R_1} = V\left[\frac{R_1}{\sum_{i=1}^{n} R_i}\right]$$

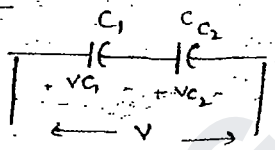
(ii) L:-



$$V_{L_1} = V\left[\frac{L_1}{L_1+L_2}\right]$$

$$V_{L_2} = V\left[\frac{L_2}{L_1+L_2}\right]$$

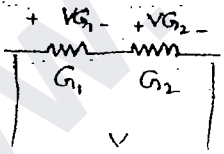
(iii) C:



$$V_{C_1} = V\left[\frac{C_2}{C_1+C_2}\right]$$

$$V_{C_2} = V\left[\frac{C_1}{C_1+C_2}\right]$$

(iv) G:

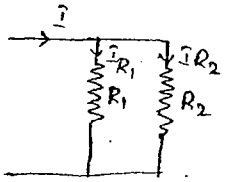


$$V_{G_1} = V\left[\frac{G_2}{G_1+G_2}\right]$$

$$V_{G_2} = V\left[\frac{G_1}{G_1+G_2}\right]$$

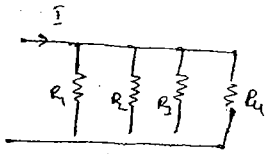
Current division Rule:-

R



$$I_{R_1} = I\left[\frac{R_2}{R_1+R_2}\right]$$

$$I_{R_2} = I\left[\frac{R_1}{R_1+R_2}\right]$$



$$I_{R_3} = I\left[\frac{R_1R_2R_4}{R_1R_2R_3 + R_2R_3R_4 + R_3R_4R_1 + R_4R_1R_2}\right]$$

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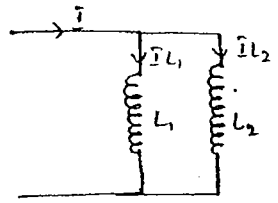
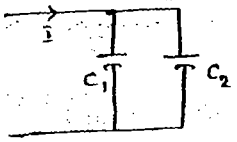
L



$$I_{L_1} = I\left[\frac{L_2}{L_1+L_2}\right]$$

$$I_{L_2} = I\left[\frac{L_1}{L_1+L_2}\right]$$

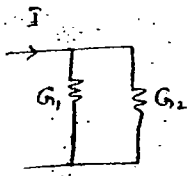
C



$$I_{C_1} = I\left[\frac{C_1}{C_1+C_2}\right]$$

$$I_{C_2} = I\left[\frac{C_2}{C_1+C_2}\right]$$

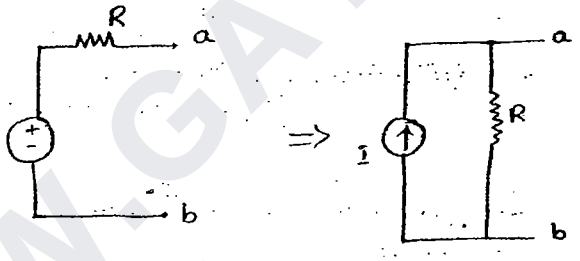
G



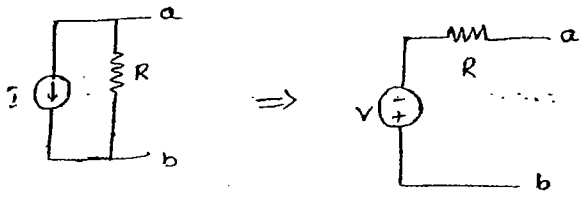
$$I_{G_1} = I\left[\frac{G_1}{G_1+G_2}\right]$$

$$I_{G_2} = I\left[\frac{G_2}{G_1+G_2}\right]$$

Source Transformation Technique:-



$$I = \frac{V}{R}$$



$$\boxed{V = IR}$$

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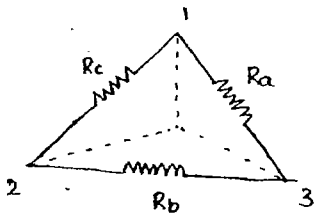
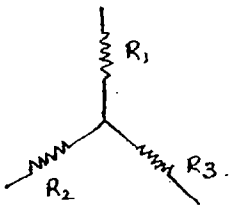
WWW.GATENOTES.IN

→ Δ − Y [Delta to Star transformation]





$$R_1 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_b}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

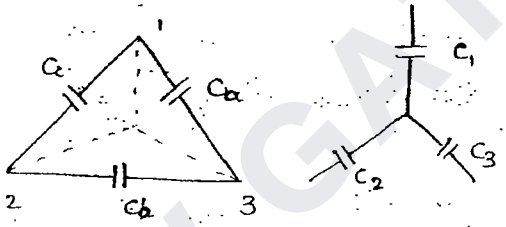
for Inductor:

$$L_1 = \frac{L_a L_c}{L_a + L_c + L_b}$$

$$L_2 = \frac{L_c L_b}{L_a + L_b + L_c}$$

$$L_3 = \frac{L_a L_b}{L_a + L_b + L_c}$$

for Capacitance:



$$C_1 = \frac{\frac{1}{C_a} \cdot \frac{1}{C_c}}{\frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C_c}}$$

$$C_2 = \frac{\frac{1}{C_c} \cdot \frac{1}{C_b}}{\frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C_c}}$$

$$C_3 = \frac{\frac{1}{C_a} \cdot \frac{1}{C_b}}{\frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C_c}}$$

for Conductance: G_1

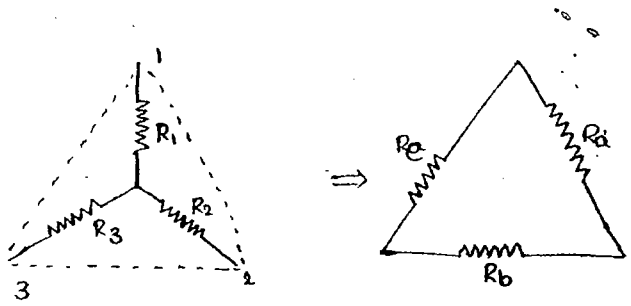
$$G_1 = \frac{\frac{1}{G_b} \cdot \frac{1}{G_c}}{\frac{1}{G_a} + \frac{1}{G_b} + \frac{1}{G_c}}$$

$$G_2 = \frac{\frac{1}{G_b} \cdot \frac{1}{G_c}}{\frac{1}{G_a} + \frac{1}{G_b} + \frac{1}{G_c}}$$

$$G_3 = \frac{\frac{1}{G_a} \cdot \frac{1}{G_b}}{\frac{1}{G_a} + \frac{1}{G_b} + \frac{1}{G_c}}$$

WWW.GATENOTES.IN

Y - Δ (star to delta)



$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = R_1 + R_2 + \frac{R_1R_2}{R_3}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = R_1 + R_3 + \frac{R_3R_1}{R_2}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \frac{R_1R_2}{R_3} + R_3 + R_1$$

WWW.GATENOTES.IN

WWW.GATENOTES.IN

Ratings / Specifications :-

Ratings are Max^m allowable Safe values for continuous operation of System

Typical electrical Equipment & Components will have voltage Rating, Current Rating, Continuous power Ratings.

- Battery also have power Rating Given in Ampere hour.

Cell phones – 1500 mAhr

Bus : 40 A hr

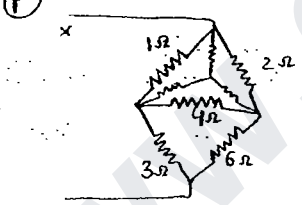
40 A x 1 hr

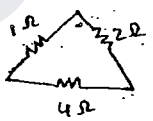
20 A x 2 hr

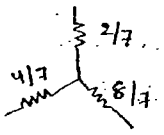
1 A x 40 hr

Pencil cell – 500 mAhr

(P)

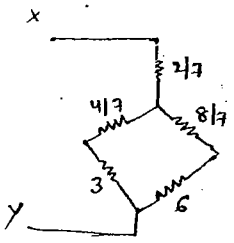






3

$4 \| 8 = \frac{4 \times 8}{12} = 8/3$



(Balance Bridge 4Ω is neglect)

$\frac{4}{7} + 3 \rightarrow \frac{25}{7}$

$\frac{8}{7} + 6 \rightarrow \frac{50}{7}$

$\frac{25}{7} \| \frac{50}{7} \Rightarrow \frac{\frac{25}{7} \cdot \frac{50}{7}}{\frac{75}{7}} = \frac{25 \times 50}{7 \times 75_3}$

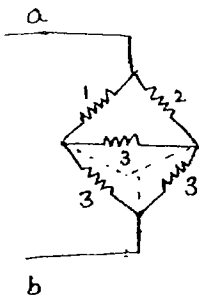
$\frac{50}{21}$

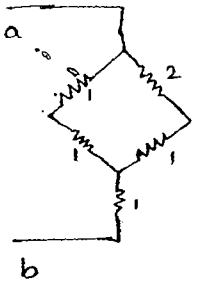
$\frac{50}{21} + \frac{2}{7} = \frac{50+6}{21} = \frac{56}{21} \frac{8}{3}$

$R_{xy} = 8/3$

WWW.GATENOTES.IN

(P)



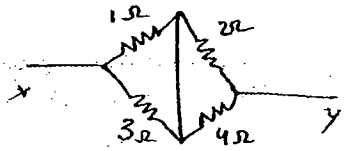


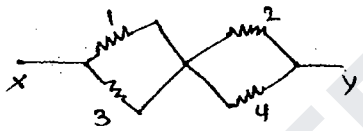
$\Rightarrow \frac{2\times 3}{5}$ $\quad \frac{2}{3}+1$

$\frac{4}{3}+1 =$

$\frac{6}{5}+1 = 11/5$

(P)





$R_{xy} = \frac{(1\|3)+(2\|4)}{}$

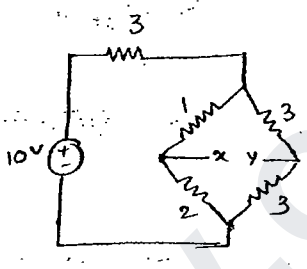
$= \frac{3}{4}+\frac{4}{3} = \frac{25}{12}\ \Omega$

(P)

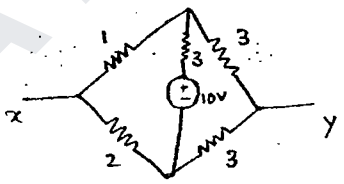
$R_{ab} = 3\|7$

$= 2.1\ \Omega$

(P)



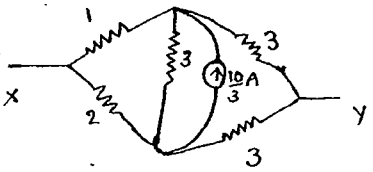
Ad R_{xy} =



$V = IR$

$I = \frac{V}{R}$

$= \frac{10}{3}$



(A) is open ckt

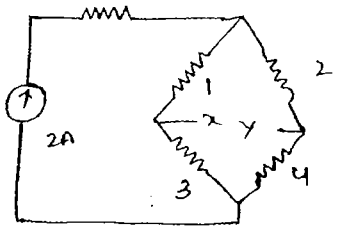
$\frac{2\times 3}{5}+1$

$= 11/5$

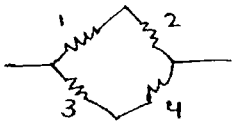
$= 11/5$

WWW.GATENOTES.IN

(P)

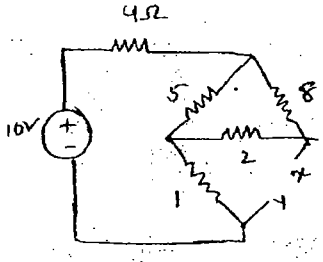


Current Source open ckt

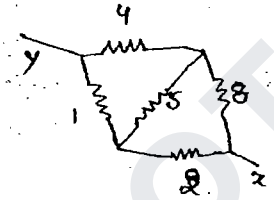


= 2·1Ω

(P)



Voltage Short ckt



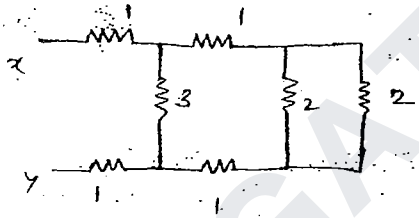
Bridge Balance

Remove 5

12 || 3

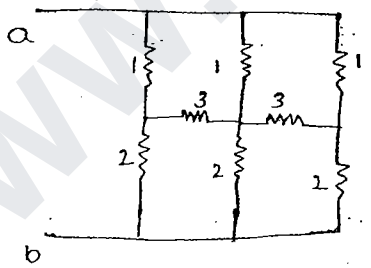
$\frac{12 \times 3}{15} = 36/15$

(P)

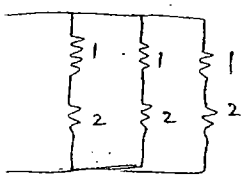


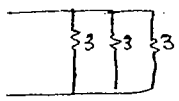
$R_{xy} = 7/2$

(P)



R_{ab} =





$\Rightarrow 9/2$

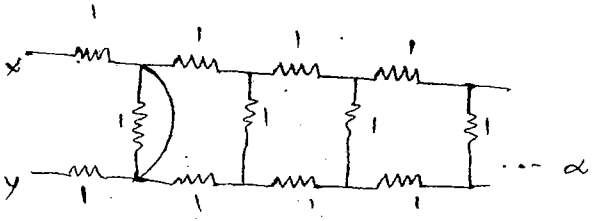
1Ω

$\frac{3.7}{6.2}$

$\frac{3}{2} + 3$

$\frac{9}{2}$

(P)



R_{xy} = 2Ω

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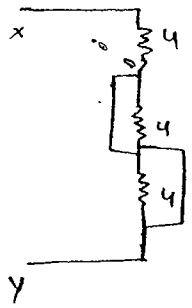
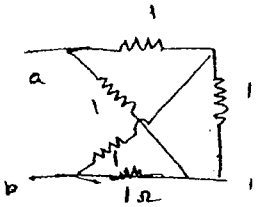
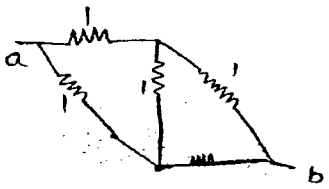
(P)



$R_{xy} = 4\Omega$

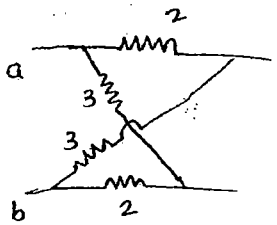
(P)

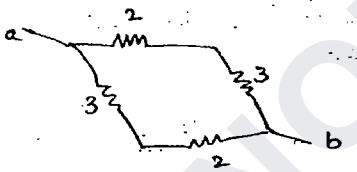




$2 \| 2 = 1\Omega$

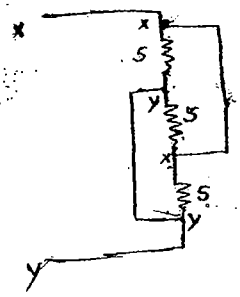
(P)

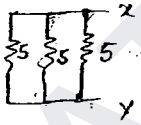




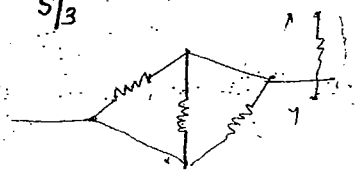
$5 \| 5 = 2.5\Omega$

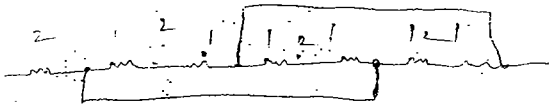
(P)



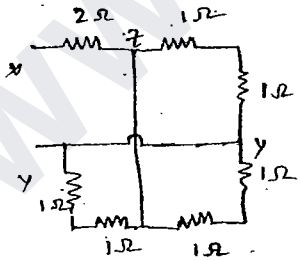


$R_{xy} = 5/3$

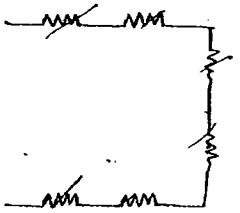




(P)

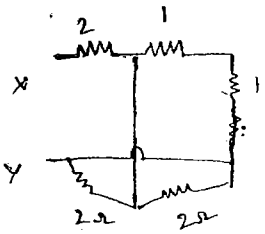


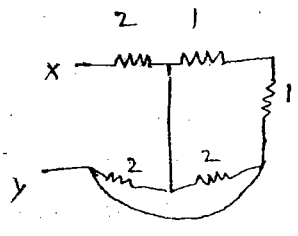
$\Rightarrow$

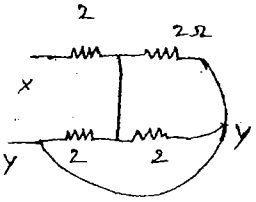


$2 + \frac{2 \times (1+1)}{2+1+2}$

$= 8/3$







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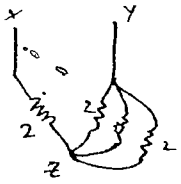
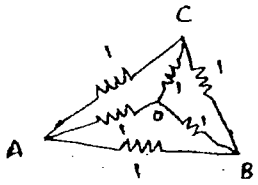
above problem

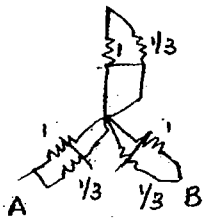


$2+\frac{2}{3} = 8/3$

$\frac{1+\frac{1}{3}}{1/3}$

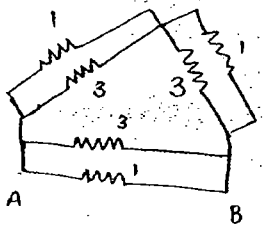
(P)



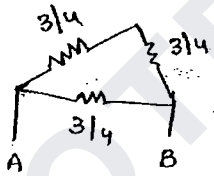


$\Rightarrow \frac{\frac{1}{3}}{1+1/3} = \frac{\frac{1}{3}}{4/3} = \frac{1}{4}$ $R_{AB} = \frac{1}{4}+\frac{1}{4}$

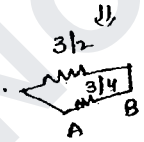
$= 1/2$



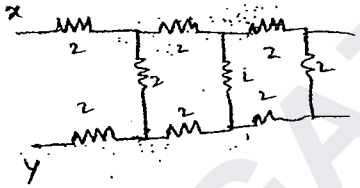
$A_{AB} = ?$



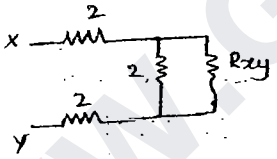
$= \frac{9/8}{\frac{3+6}{4}} = 1/2$



(P)



$R_{xy} =$

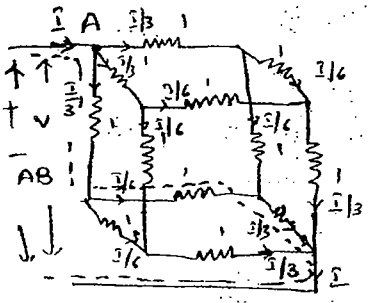


$R_{xy} = (4+(2 \| R_{xy}))$

$R_{xy} = 4 + \frac{2 \cdot R_{xy}}{2+R_{xy}}$

$(R_{xy})(2+R_{xy}) = 8+6R_{xy}$

$R_{xy}^2 = 4R_{xy} + 8$



$R_{AB} = ?$

$-V + \frac{I}{3}[1] + \frac{I}{6}[1] + (\frac{I}{3})1 = 0$

$V = I\left[\frac{1}{3}+\frac{1}{6}+\frac{1}{3}\right]$

$= I\left[5/6\right]$

$R_{AB} = \frac{V}{I} = 5/6\ \Omega$

$R_{AB} = \frac{5r}{6}$

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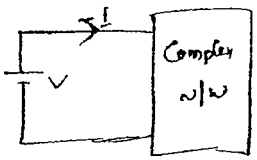
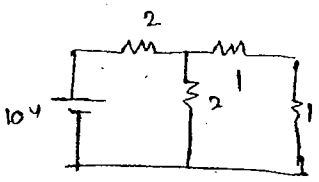
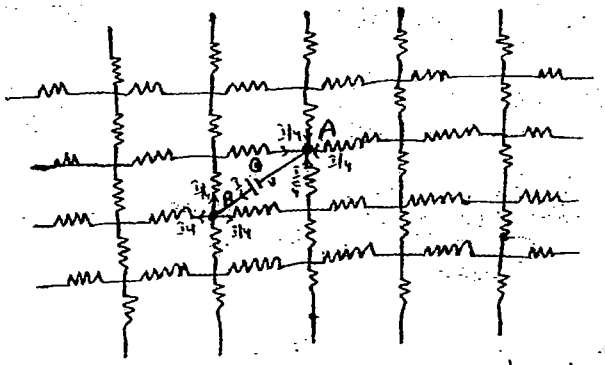
Concept of Multimeter:-



$R_{practical} = \frac{V}{I}$



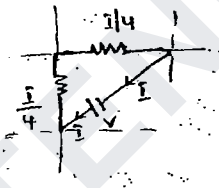
(P)



$r = 3\Omega$

R_{AB}

Sol ①



$-V + \frac{I}{4}(3) + \frac{I}{4}(3) = 0$

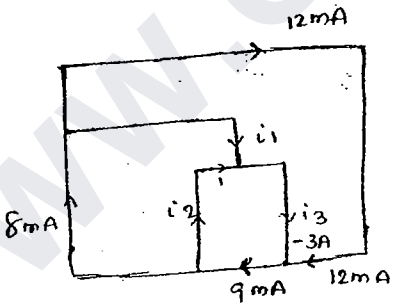
$V = I\left[\frac{3}{4} + \frac{3}{4}\right]$

$V = I\left[3/2\right]$

$\frac{V}{I} = 3/2\ \Omega$

$R_{AB} = r/2$

(P)



$i_1 = -4mA$

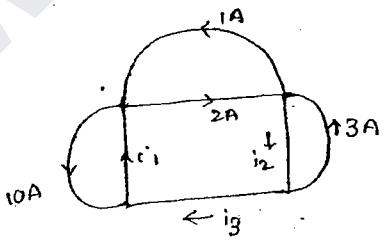
$i_2 = 1mA$

$i_3 = -3A$

$i_1 + 1mA = -3$

$i_1 = -4mA.$

(P)



$i_1 = 11A$

$i_2 = 5 - 1 = 4A$

$i_3 = 1A$

$i_1 = 10 + i_3$

$i_2 = 4A$

$i_3 = 4 - 3$

$= 1A$

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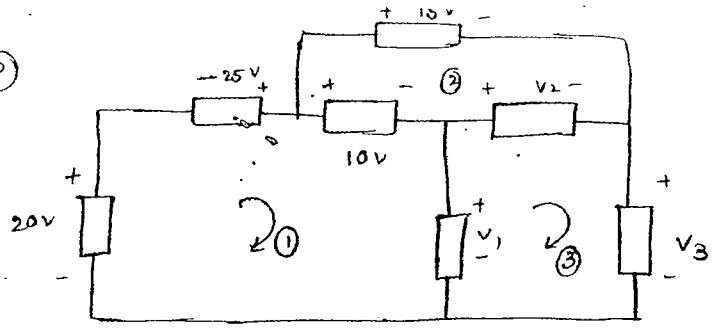
WWW.GATENOTES.IN

(P)



① $-20 - 25 + 10 + V_1 = 0$ $\quad V_1 = 35V$

② $15 - V_2 - 10 = 0$

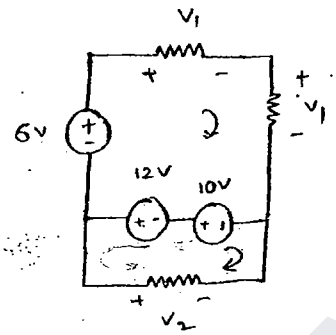
$V_2 = 5V$

③ $V_3 - V_1 + V_2 = 0$

$V_3 - 35 + 5 = 0$

$V_3 = 30V$

(P)



$-6 + V_1 + V_1 - 10 - 12 = 0$

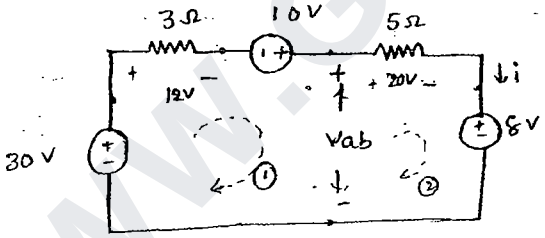
$2V_1 = 28$

$V_1 = 14$

$-V_2 + 12 + 10 = 0$

$V_2 = 22V$

(P)



5 nodes

5 Branches

Eq's 1

Vab = ?

1st total loop

$-30 + 3i - 10 + 5i + 8 = 0$

$8i = 32$

$i = 4A$

KVL ①

$-30 + 12 - 10 + Vab = 0$

$Vab = 28$

KVL ②

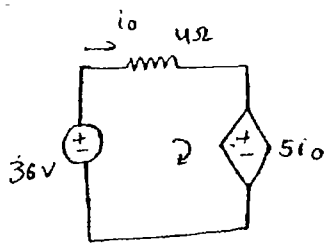
$-Vab + 20 + 8 = 0$

$Vab = 28$

WWW.GATENOTES.IN

WWW.GATENOTES.IN

(P)

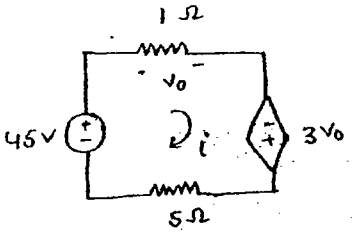


$-36 + 4i_o + 5i_o = 0$

$9i_o = 36$

$i_o = 4$

(P)



$-45 + i - 3V_o + 5i = 0$

$6i = 3V_o + 45 \quad -①$

$V_o = 1 \times i$

$V_o = i \quad -②$

② in ①

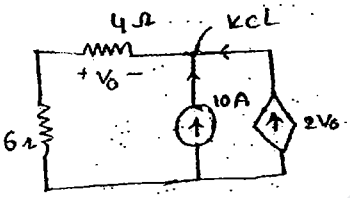
$6i = 3i + 45$

$3i = 45$

$i = 15$

$V_o = 15V$

(P)



KCL at node

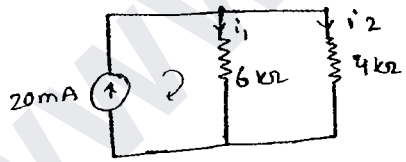
$10 + 2V_o$

$V_o = -[10 + 2V_o]\,4$

$V_o = -40 - 8V_o \qquad 9V_o = -40$

$V_o = -40/9\ V$

(P)



$i_1 = ?$

$i_2 = ?$

$i_1 + i_2 = 20\ mA$

① $\Rightarrow$ 20 mA

$i_1 = 20mA\left[\frac{4}{10}\right] = 8\ mA$

$i_2 = 12\ mA = (20-8)\ mA$

$12\ mA$

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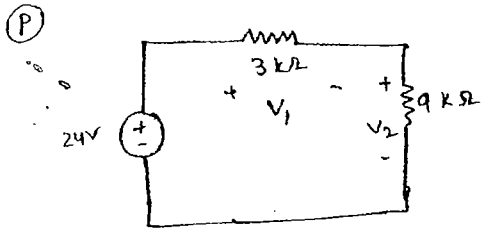
WWW.GATENOTES.IN

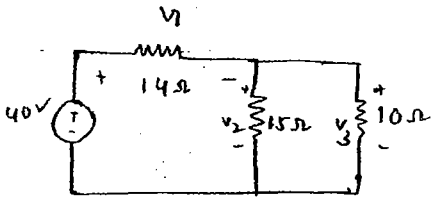
(P)



$V_1 = 24\left[\frac{3}{3+9}\right] = 6V$

$V_2 = 24\left[\frac{9}{12}\right] = \underline{18V}$

(P)



$V_1 = 40\left[\frac{14}{14+6}\right]$ $15 \| 10$

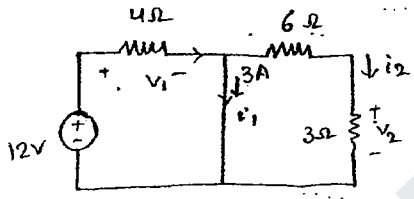
$= 28V$

$V_2 = 40 - 28$ $V_2 = 28\left[\frac{10}{15}\right]$

$= \underline{12V}$ $12V$

parallel $(V_2 = V_3)$ $V_3 = \underline{12V}$

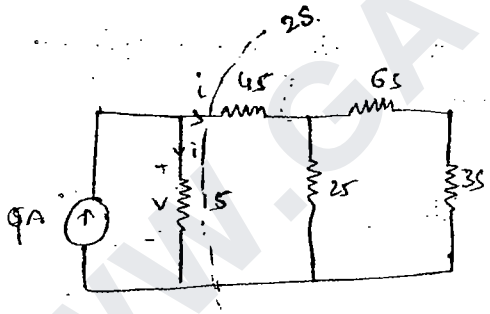
(P)



$V_1 = 12V$ $i_1 = 3A$

$V_2 = 0$ $i_2 = 0$

(P)



opposite

$i = 9\left[\frac{2}{2+1}\right] = 6A$

$V_1 = 5\left[\frac{62/11}{15}\right] = \frac{5 \times 62}{11}$ $V = iR$

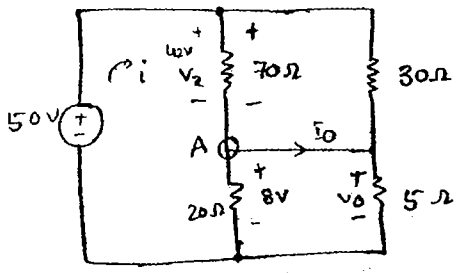
$= 6 \times 3 \times \frac{1}{15} = 3V$

$i = 5 - \frac{62 \times 5}{11}$ 5[

3. Sources

WWW.GATENOTES.IN

(P)



nodes = 3

$70 \| 30 = 21\ \Omega$

$20 \| 5 = 4\ \Omega$

$50\left[\frac{21}{21+4}\right] = 42$

$V_0 = 8V$

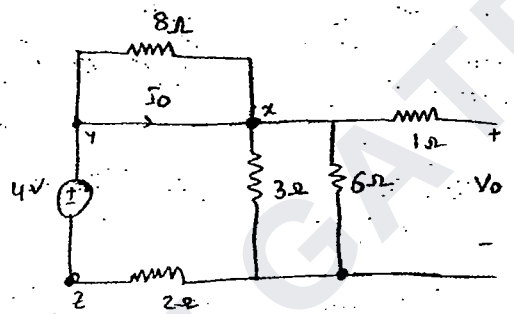
$i = \frac{50}{4}$

at node A → KCL

$\frac{42}{70} - I_0 - \frac{8}{20} = 0$

$I_0 = \frac{42}{70} - \frac{8}{20} = 0.2\ A$

(P)



Nodes = 3

branches = 5

Eq's 5−3+1

3 meshes

1Ω is open

$6 \| 3 = \frac{18}{9} = 2$ & 2 Series 4Ω

4V & 4Ω $i = 1\ A$

$i_0 = 1A$

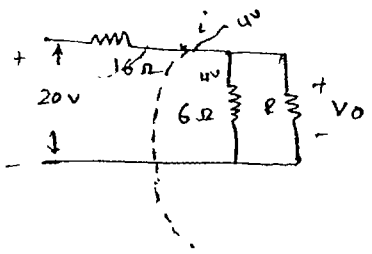
$i_{3\Omega} = 1\left[\frac{6}{9}\right] = 2/3$

$i_{6\Omega} = 1/3$ $\quad V_0 = \frac{1}{3} \times 6 = 2V$

WWW.GATENOTES.IN

(P)

if $V_o = 4V$ then $R =$



① across 6Ω - 4V

across 16Ω - 16V

$i = 1A$ ✓

$i_{6\Omega} = +x$

$= \frac{4}{6} \frac{2}{3}$ $i = 1V$

② $4V = R\left[1\left[\frac{6}{6+R}\right]\right]$ Total Current

$24 + 4R = 6R$

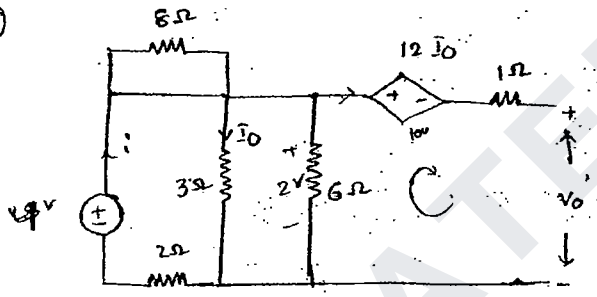
$2R = 24$

$R = 12\Omega$

$i_R = 1 - 2/3 = 1/3$

$R = \frac{4}{1/3} = 12\Omega$

(P)



8Ω is Short $3 \| 6 + 2 = 4\Omega$

$i = \frac{4}{4} = 1A$ ✓

$i_o = 1\left[\frac{6}{6+3}\right] = 2/3$

$V_{3\Omega} = 2V = V_{6\Omega}$

in 1Ω the Current (0)

$-2 + 12\left(\frac{2}{3}\right) + V_o = 0$

$V_o = 6$

$\frac{6 \times 3}{6+3} = \frac{2}{9\Omega}$

$\frac{5}{4}$

$\frac{5}{4} \times \left(\frac{6}{9}\right)$

$\frac{5}{6}$

$\frac{12 \times 5}{6}$

10

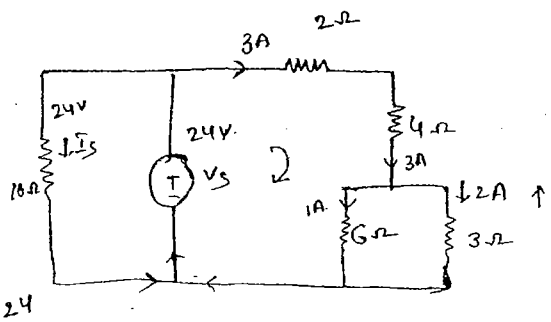
$12 \times I_o$

$-2 + 12\left(\frac{2}{3}\right) + V_o = 0$

$V_o = 6$

$I = 6V$

(P)



$V_s =$

$I_s =$

$V_{3\Omega} = 2 \times 3 = 6V$

$V_{6\Omega} = 6V$

$i_{6\Omega} = 1A$

$I_s = \frac{24}{10} = 2.4A$

$KVL = -V_s + 6 + 12 + 6 = 0$

$V_s = 24V$

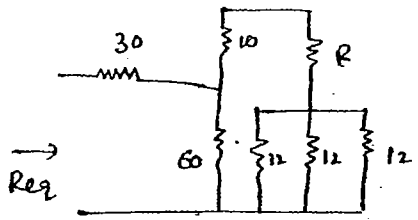
$I_s = 2.4A$

$i_{4\Omega} = 1 + 2 = 3A$

WWW.GATENOTES.IN

(P)

If $R_{eq} = 50\Omega$ then R:



$12 \| 12 \| 12 = 4\Omega$

$$R_{eq} = 50 = 30 + (60 \| 04+R)$$

$$50 = 30 + \frac{60(14+R)}{74+R}$$

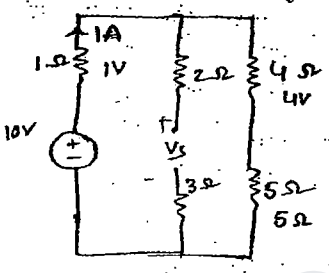
$$20(74+R) = 60(14+R)$$

$$74+R = 42+3R$$

$$2R = 32$$

$$R = 16\Omega \checkmark$$

(P)



$V_s = ?$

$$-V_s + 4 + 5 = 0$$

$$V_x = 9V$$

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Methods of Analysis

→ Mesh Analysis – ohm's law + KVL (Current Sources more than voltage source)

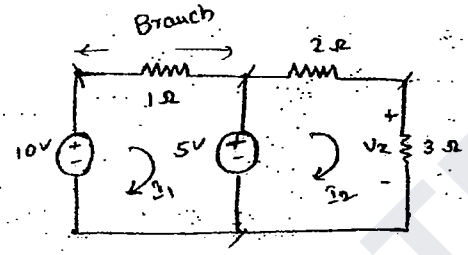
→ Nodal Analysis – ohm's law + KCL

(Voltage Source are more the C.S)

DC Circuit $f = 0Hz$

L – S.C

C – O.C



V_x = ? use Mesh & Nodal analysis.

Nodes = 4

Branch = 5

branch – Nodes + 1

5 – 4 + 1 = 2

Mesh:-

→ No of Meshes = 2 (Assumes the no of Mesh Currents)

→ Both Clockwise

→ KVL & Solve to find mesh Currents

$-10 + I_1 + 5 = 0$

$I_1 = 5A$

$-5V + 2I_2 + 3I_2 = 0$

$I_2 = 1\ A$

$V_x = 1 \times 3 = 3$ Volts

Node:-

→ Identify No of Nodes

→ Neglect Series nodes

→ Consider one node as Reference node.

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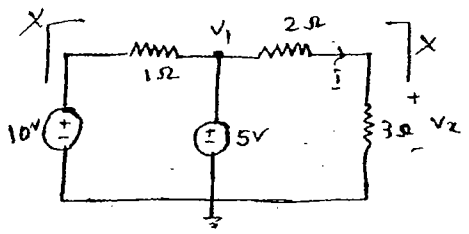
(3)

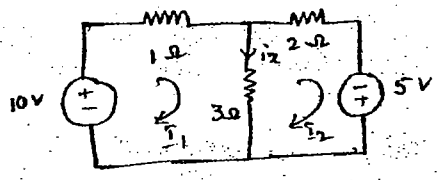


$V_1 = 5V$ ✓

$I = \frac{5-0}{5} = 1A$

$V_2 = 3 \times 1 = 3V$

(4)



Mesh

① $-10 + I_1 \times 1 + 3(I_1 - I_2) = 0$

$4I_1 - 3I_2 = 10$

② $3(I_2 - I_1) + 2I_2 - 5 = 0$

$-3I_1 + 5I_2 = 5$

$4I_1 - 3I_2 = 10$ } ×3

$-3I_1 + 5I_2 = 5$ } ×4

$I_2 = \frac{50}{11} A$

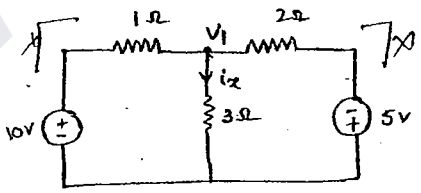
$4I_1 = 10 + 3\left(\frac{50}{11}\right)$

$I_1 = 24.6A$ $i_2 = \frac{65}{11} A$

$i_2 = \frac{65}{11} - \frac{50}{11}$

$= 15/11$ A

Nodal :-



$\frac{V_1 - 10}{1} + \frac{V_1}{3} + \frac{V_1 + 5}{2} = 0$

$\frac{11V_1}{6} = 10 - \frac{5}{2} = \frac{15}{2}$

$V_1 = \frac{15}{2} \times \frac{6}{11} 3 = 45/11$

$I_{2\Omega} = \frac{V_1}{3}$

$= \frac{45}{11}\Big|_3 = 15/11$ A

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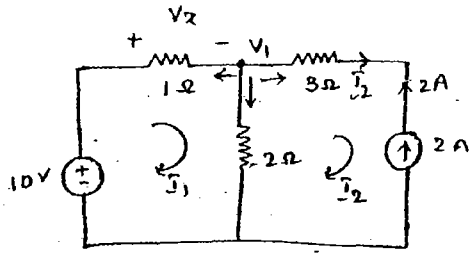
Model Number -③ :-



Mesh $-10 + I_1 + 2(I_1 - I_2) = 0$ — ①

$2(I_2 - I_1) + 3I_2$ $\quad I_2 = -2A$

$3I_1 - 2I_2 = 10$

$3I_1 + 4 = 10$

$I_1 = 2A$

$V_x = iR$

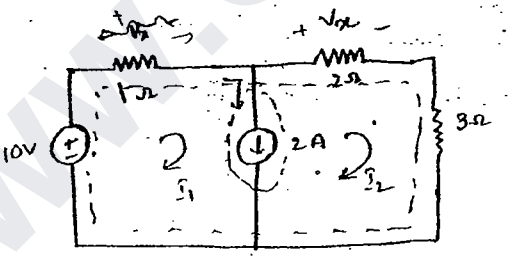
$= 2 \times 1 = 2\Omega$

Nodel :-

$$\frac{V_1 - 10}{1} + \frac{V_1}{2} - 2 = 0$$

$\frac{3V_1}{2} = 12$ $\quad V_1 = 8V$ $\quad V_x = 10 - 8 = 2V$

Model Number -④:



Super mesh

Current & voltage are in same loop

$-10 + I_1 + 2I_2 + 3I_2 = 0$ — ③

$I_1 + 5I_2 = 10$ — ①

$I_1 - I_2 = 2$ — ②

① - ②

$6I_2 = 8$

$I_2 = 4/3$

$V_x = -4/3 \times 2$

$= -8/3 V$

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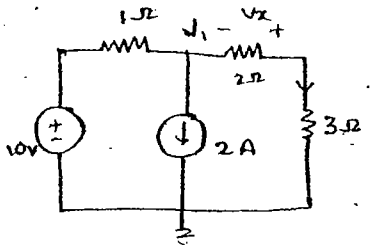
$$\frac{V_1-10}{1} + 2 + \frac{V_1}{5} = 0$$



$$\frac{6V_1}{5} = 8 \qquad V_1 = \frac{40}{6} = 20/3$$

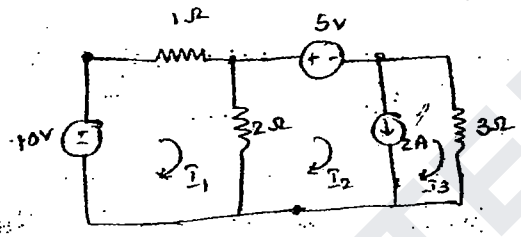
$$i_{3\Omega} = \frac{20}{3\times 5} = 4/3\ A$$

polarity V_x is −ve So $V_x = -2\times\frac{4}{3} = -\frac{8}{3}V$

27/4/12

Model Number-5:

Mesh



find the power delivered by 5V Source using mesh, nodal

$$I_1 + 2(I_1 - I_2) - 10 = 0$$

$$3I_1 - 2I_2 = 10$$

$$I_1 = \frac{10+2I_2}{3}$$

$$I_2 = \frac{10-3I_1}{-2} = \frac{3I_1-10}{2}$$

$$= \frac{3\left(14+\frac{2}{11}-10\right)}{2}$$

$$= 23/11\ A.$$

$$I_2 - I_3 = 2A$$

$$I_2 = 2 + I_3$$

$$\frac{3I_1-10}{2} = 2 + I_3$$

$$3I_1 - 10 = 4 + 2I_3$$

$$3I_1 = 14 + 2I_3$$

$$I_1 = \frac{14+2I_3}{3}$$

$$I_1 = \frac{14+\frac{2}{11}}{3} =$$

$$2(I_2 - I_1) + 5 + 3I_3 = 0$$

$$-2I_1 + 2I_2 + 3I_3 + 5 = 0$$

$$-2I_1 + 2(2+I_3) + 3I_3 + 5 = 0$$

$$-2I_1 + 4 + 2I_3 + 3I_3 + 5 = 0$$

$$-2I_1 + 5I_3 + 9 = 0$$

$$-2\left(\frac{14+2I_3}{3}\right) + 5I_3 + 9 = 0$$

$$-2(14+2I_3) + 15I_3 + 27 = 0$$

$$11I_3 = 1$$

$$I_3 = 1/11$$

power delivered by 5V Source $= -5\times\frac{23}{11} = -\frac{115}{11}V$

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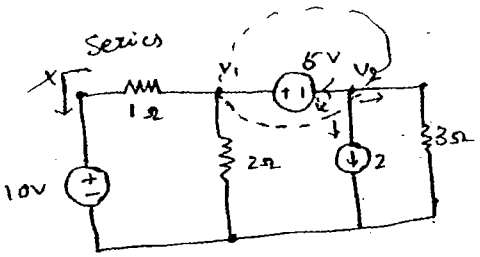
Node:



Super node

KVL

Current leaving +ve

$$\frac{V_1 - 10}{1} + \frac{V_1}{2} + 2 + \frac{V_2}{3} = 0$$

$$9V_1 + 2V_2 = 48$$

$$V_1 - V_2 = 5 \qquad V_1 = \frac{53}{10} \quad V_2 = 3/10$$

$$V_1 = 58/11$$

$$V_2 = 3/11$$

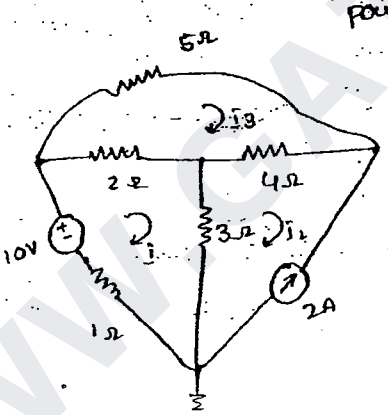
at node V_2

$$i_x + \frac{1}{11} + 2 = 0$$

$$\therefore i_x = -23/11\ A$$

power deliver by 5V source $\frac{-115}{11}$ watts

(P)



write Matrix form Eqn's

Mesh eqn's.

$$I_1 - 10V + 2(I_1 - I_3) + 3(I_1 - I_2) = 0$$

$$6I_1 - 3I_2 - 2I_3 - 10V = 0 \quad \text{--- (1)}$$

$$3(I_2 - I_1) + 4(I_2 - I_3) + 2 = 0$$

$$-3I_1 + 7I_2 - 4I_3 = -2$$

$$I_2 = -2A \quad \text{--- (2)}$$

$$5I_3 + 2(I_3 - I_1) + 4(I_3 - I_2) = 0$$

$$-2I_1 - 4I_2 + 11I_3 = 0 \quad \text{--- (3)}$$

$$\begin{bmatrix} 6 & -3 & -2 \\ -3 & 7 & -4 \\ -2 & -4 & 11 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -2 \\ 0 \end{bmatrix}$$

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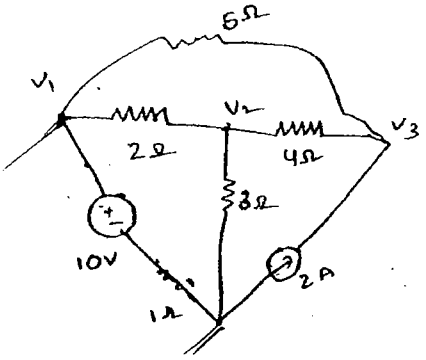
Node1:-



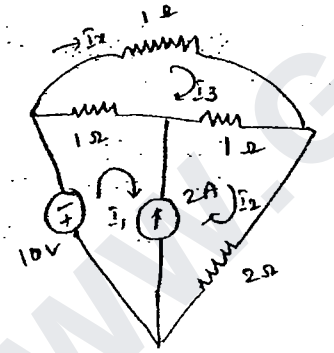
$$\frac{V_1-10}{1}+\frac{V_1-V_2}{2}+\frac{V_1-V_3}{5}=0 \qquad 17V_1-5V_2-V_3=100$$

$$\frac{V_2-V_1}{2}+\frac{V_2}{3}+\frac{V_2-V_3}{4}=0 \qquad -6V_1+13V_2-3V_3=0$$

$$\frac{V_3-V_1}{5}-2+\frac{V_3-V_2}{4}=0 \qquad -4V_1-5V_2+9V_3=40$$

$$\begin{bmatrix}17 & -5 & -1\\ -6 & 13 & -3\\ -4 & -5 & 9\end{bmatrix}\begin{bmatrix}V_1\\V_2\\V_3\end{bmatrix}=\begin{bmatrix}100\\0\\40\end{bmatrix}$$

(P)



Find i_x using Mesh & Nodal.

$10+i_1+2i_2\times-i_1$

$10+(i_1-i_3)+(i_2-i_3)+2i_1=0$

$4i_1+2i_2=-10$ — ① $\quad i_1+3i_2-2i_3=10$

$i_3+(i_3-i_2)+(i_3-i_1)=0$

$-i_1-i_2+3i_3=0$

$i_1=-i_2+3i_3$

$i_2-i_3+2i_2+10+i_2-i_3=0$

$4i_2-2i_3=-10$ — ②

$4(-i_2+3i_3)-2i_3=-10$

$-6i_2+10i_3=-10$

$i_2=\frac{-15}{4}$ $\quad i_3=-5/2$

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$$10 + [I_1 - I_3] + [I_2 - I_3] + 2I_2 = 0$$

$$I_1 + 3I_2 - 2I_3 = -10 \quad -(1)$$

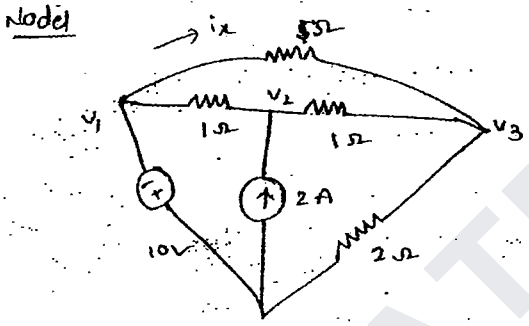
$$-I_1 + I_2 = 2 \quad -(2)$$

$$I_3 + [I_3 - I_2] + [I_3 - I_1] = 0$$

$$-I_1 - I_2 + 3I_3 = 0 \quad -(3)$$

Solve (1) (2) (3) $I_3 = -3V$

Nodel



$V_1 = -10$

$$-2 + \frac{V_2 - V_1}{1} + \frac{V_2 - V_3}{1} = 0$$

$$\frac{V_3 - V_2}{1} + \frac{V_3}{2} + \frac{V_3 - V_1}{1} = 0$$

$$V_1 = -10 \quad -(1)$$

$$-V_1 + 2V_2 - V_3 = 2 \quad -(2)$$

$$-2V_1 - 2V_2 + 5V_3 = 0 \quad -(3)$$

$$2V_2 - V_3 = -8$$

$$-2V_2 + 5V_3 = -20$$

$$4V_3 = -28$$

$$V_3 = -7$$

$$i_x = \frac{V_1 - V_3}{1} = \frac{-10 - (-7)}{1} = -3A$$

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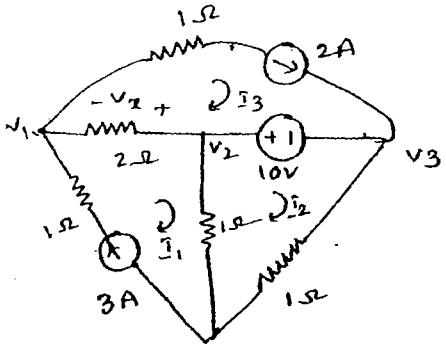
(P)



find V_x using mesh nodal

mesh

$I_1 = 3A$ — ① $\quad I_2 + (I_2 - I_1) + 10 = 0$

$-I_1 + 2I_2 + 10 = 0$ — ②

$I_3 = 2A$

$-3 + 2I_2 = -10$

$2I_2 = -7$

$I_2 = -7/2$

$2(I_3 - I_1) = V_x$

$2(2-3) = V_x$

$V_x = -2$

Nodal

$-3 + \frac{V_1 - V_2}{2} + 2 = 0$ — ①

V_2

$\frac{V_1 - V_2}{2} = -1$

$V_1 - V_2 = -2$

$V_1 = 8V$

$V_2 = 10$

$V_3 = -10$

$3 + \frac{V_1 - 10}{2} = 2$

$V_1 = -2$

$V_1 = 8$

$V_2 = 10$

$\frac{V_1 - V_2}{2} = 2$

$8 - V_2$

$-3 + \frac{V_1 - V_2}{2} + 2 = 0$

$V_1 - V_2 = 2$

$V_2 - V_1 = V_x = -2V$

$\frac{V_2 - V_1}{2} + \frac{V_2}{2} + \frac{V_3}{1} - 2 = 0$

$-V_1 + 3V_2 + 2V_3 = 4$

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$$V_2 - V_3 = 10 \quad -③$$

③ × 2 $\quad 2V_2 - 2V_3 = 20$

$$-V_1 + 5V_2 = 24$$

① + ④ $\Rightarrow 4V_2 = 26$

$$V_2 = 13/2$$

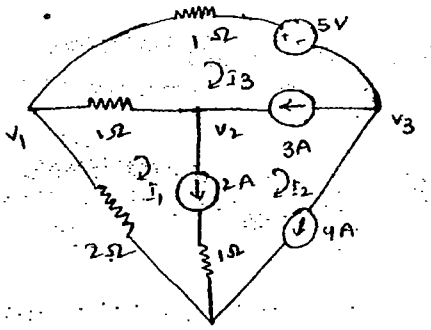
$$V_1 = 2 + 13/2$$

$$V_1 = 17/2$$

$$V_x = V_2 - V_1$$

$$= \frac{13}{2} - \frac{17}{2} = -\frac{2}{2}\ V$$

(P)



power deliver by 5V source.

direct inspection KCL

$$I_{5V} = 3 + 4 = 7A$$

power = $7 \times 5 = -35$ watts

Mesh

$$2I_1 + 1(I_1 - I_3) + (I_2 - I_1)1 + (I_3 - I_1)1 + I_3 + 5 = 0$$

$$I_1 - I_2 = 2A \qquad I_2 = 4A \qquad I_3 - I_2 = 3A$$

$$I_1 = 6 \qquad I_3 = 7A \qquad I_2 = 4$$

$$I_3 \cdot V = 7 \times 5 = 35$$

Reverse $= -35$

Nodal

$$\frac{V_1}{2} + \frac{V_1 - V_2}{1} + \frac{V_1 - V_3 - 5}{1} = 0$$

$$\frac{V_2 - V_1}{1} + 2 - 3 = 0$$

$$\frac{V_3 + 5 - V_1}{1} + 3 + 4 = 0$$

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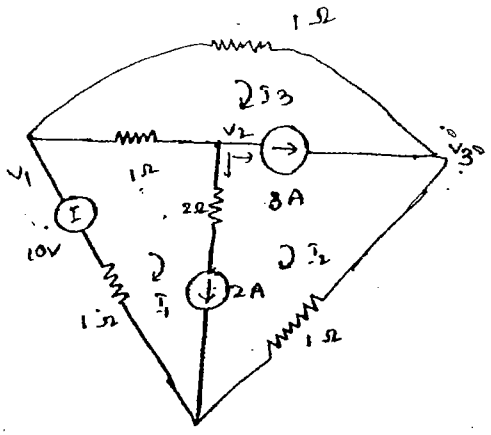
(P)



power deliver by 2 A

Current source by mesh & nodel

Mesh

$I_1 - I_2 = 2A$ —(1)

$I_2 - I_3 = 3A$ —(2)

~~$I_1 - 10V + I_1 - I_3 + 2 I_1 - I$~~

$I_1 - 10 + I_3 + I_2 = 0$ —(3)

$I_1 = 17/3$ A

$I_2 = 11/3$

$I_3 = 2/3$ A

Nodel.

$$\frac{V_1 - 10}{1} + \frac{V_1 - V_2}{1} + \frac{V_1 - V_3}{1} = 0$$

$3V_1 - V_2 - V_3 = 10$ — (1)

$$\frac{V_2 - V_1}{1} + 2 + 3 = 0$$

$-V_1 + V_2 = -5$ — (2)

$$\frac{V_3 - V_1}{1} + \frac{V_3}{1} - 3 = 0$$

$-V_1 + 2V_3 = 3$ — (3)

(2) & (3)

$-V_1 + V_2 = -5$

$-V_1 + 2V_3 = 3$

$-V_3 = -8$

$V_3 = 8V$

$V_2 - 2V_3 = -8$

$V_2 = 2V_3 - 8$

$-V_1 + 2V_3 - 8 = -5$

$-V_1 + 2V_3$

$V_1 = 13V$

$3(13) - V_2 - 8 = 10$

$V_2 = 21$

$3V_1 - 2V_3 + 8 - V_3 = 10$

$3V_1 - 3V_3 = 2$

$-V_1 + 2V_3 = 3$

$V_1 = 13/3$ $V_3 = 11/3$

$V_2 = -2/3$

P = V I
= 2 × 4
= 8V

39
18
21

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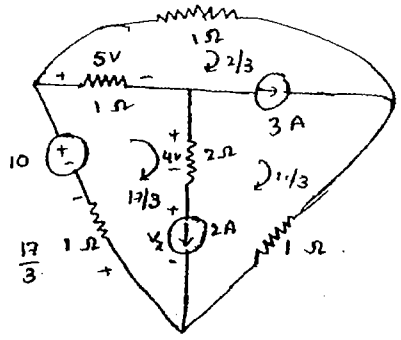
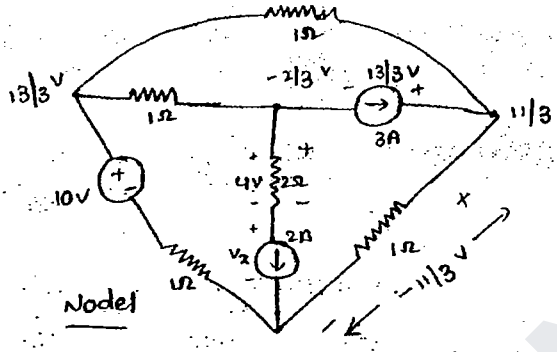


$$\frac{17}{3} - 10 + 5 + 4 + V_x = 0$$

$$V_x = -14/3.$$

power $= 2 \times +\frac{14}{3} = +28/3$



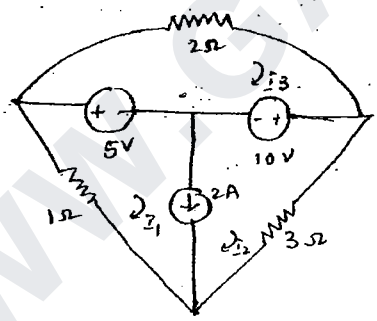
$$11/3 - (-2/3)$$

$$-V_x - 4 - 13/3 + 11/3 = 0$$

$$V_x = -14/3$$

power across 2A $= 2(-14/3) = +28/3$

(P)



power dissipated by 3Ω Resistor

Mesh

$$I_1 - I_2 = 2A$$

$$I_1: \quad I_1 + 5 - 10 + 3I_2 = 0 \qquad I_1 + 3I_2 = 5$$

$$2I_3 + 10 - 5 = 0 \qquad I_1 = 11/4 \quad I_2 = 3/4$$

$$I_3 = -5/2$$

$$P_{3\Omega} = I_2^2 R = 27/16$$

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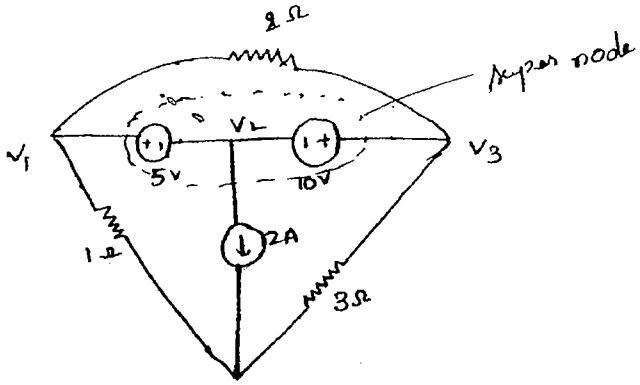
Node 1



$$\frac{V_1}{1} + 2 + \frac{V_3}{3} + \frac{V_3 - V_1}{2} + \frac{V_1 - V_3}{2} = 0$$

$2\left(\frac{2, 2, 3}{1, 3}\right)$

$$6V_1 + 12 + 2V_3 + 3V_3 - 3V_1 + 3V_1 - 3V_3 = 0$$

$$6V_1 + 2V_3 = -12 \Rightarrow 3V_1 + V_3 = -12$$

$$V_1 - V_2 = 5V \qquad V_2 - V_3 = 10$$

$$V_1 = 5 + V_2$$

$$3(5 + V_2) + V_3 = -6$$

$$3V_2 + V_3 = -21$$

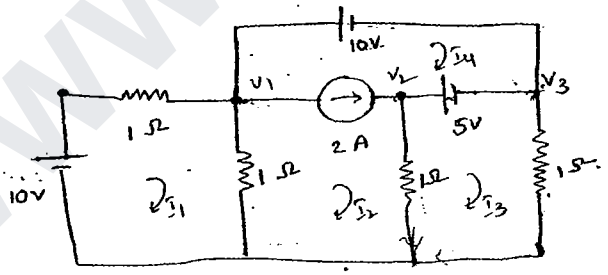
$$-V_2 + V_3 = 10$$

$$V_2 = -31/4 \qquad V_3 = 9/4$$

$$P_{3\Omega} = \frac{V_3^2}{R} = \frac{(9/4)^2}{3}$$

$$= 27/16 \text{ watts}$$

(P)



apply Mesh & Nodel Equations

Mesh

node = 5

Branches = 8

meshes = 8 − 5 + 1 = 4

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Mesh

$-10 + I_1 + (I_1 - I_2) = 0$

$I_2 - I_4 = 2A$ $\quad (I_3 - I_2) + 5V + I_3 = 0$

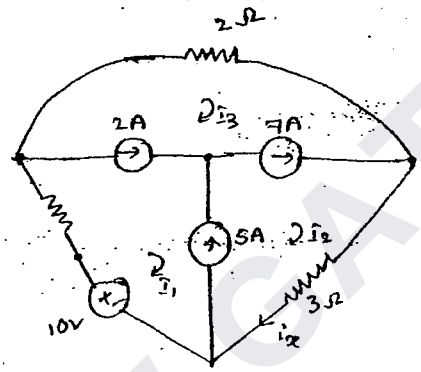
$(I_2 - I_1) + 10 - 5 + (I_2 - I_3) = 0$

Nodal

$\frac{V_1 - 10}{1} + \frac{V_1}{1} + \frac{V_1 - V_3}{2} + \frac{V_3}{1} + \frac{V_2}{1} - 2 = 0$

$V_2 - V_3 = 5 \qquad V_1 - V_3 = 10$

(P)



Find i_x by Mesh & Nodal.

Node - 5

Branches = 7

Meshes = 7 - 5 + 1

= 3

Mesh

$I_2 - I_1 = 5A$

$I_2 - I_3 = 7A$

$I_1 - I_3 = 2A$

} Cannot be Solved.

Master Equation:

$-10 + I_1 + 2I_3 + 3I_2 = 0$

$I_1 + 3I_2 + 2I_3 = 10 \quad -(4)$

$I_1 = I_2 - 5$

$I_2 - I_3 = 7$

$I_3 = I_2 - 7$

$I_2 - 5 + 3I_2 + 2(I_2 - 7) = 10$

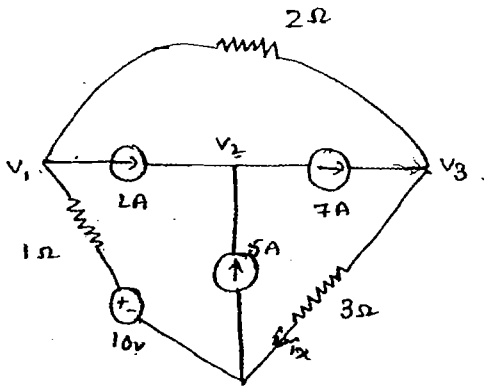
$6I_2 = 29 \qquad I_2 = 29/6 = 4.83$

$i_x = 4.83$

WWW.GATENOTES.IN

(P)

Node 1



find ix by Nodal analysis.

$$\frac{V_1 - 10}{1} + 2 + \frac{V_1 - V_3}{2} = 0 \quad -(1)$$

$$-2 - 5 + 7 = 0 \quad -(2)$$

$$\frac{V_3}{3} - 7 + \frac{V_3 - V_1}{2} = 0 \quad -(3)$$

(1) $\Rightarrow$ $3V_1 - 18 - V_3 = 0$

$$3V_1 - V_3 = 18 \quad -$$

(3) $\Rightarrow$ $5V_3 - 42 - 3V_1 = 0 \Rightarrow 3V_1 - 5V_3 = -42$

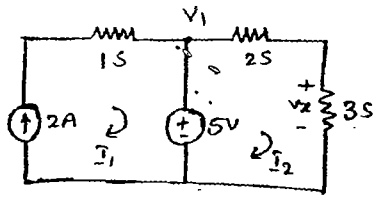
$$V_1 = 41/6 \qquad V_3 = 25/2$$

$$i_x = \frac{V_3}{3\Omega} = \frac{25/2}{3} =$$

$$i_x = \frac{25}{6} = 4.1$$

WWW.GATENOTES.IN

(P)



Find V_x mesh and loop

Nodel Analysis.

Mesh KVL $\Rightarrow$ $V = I/G$

Nodel KCL $\Rightarrow$ $I = V \cdot G$

Mesh

$I_1 = 2$ —(1)

$$-5 + \frac{I_2}{2} + \frac{I_2}{3} = 0$$

$$3I_2 + 2I_2 = 30$$

$$5I_2 = 30$$

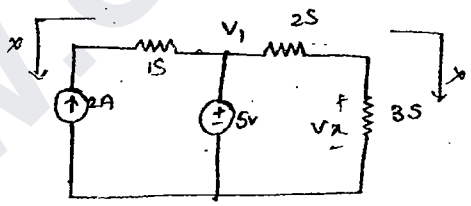
$$I_2 = 30/5 = 6$$

if $I_2 = 6A$ Voltage $V_x = I/G$

$V_x = 6/3 = \frac{2V}{=}$

$\boxed{V_x = 2V}$

Nodel:



Series Branch neglect

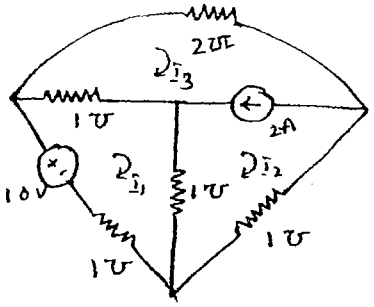
$V_1 = 5V$

$$i_{3S} = V_1 \left[2 \text{ Series } 3 \right] = V_1 \left[\frac{6}{5} \right] = 5 \times \frac{6}{5} = 6A$$

$$V_x = \frac{i_{3S}}{G} = \frac{6}{3} = \frac{2V}{=}$$

WWW.GATENOTES.IN

(P)



Find power loss in 2 Ω by using mesh & Nodel

Mesh

$$\frac{I_1}{1} - 10 + \frac{I_1}{1} + \frac{I_1 - I_2}{1} = 0$$

$$3I_1 - I_2 = 10 \quad \text{— (1)}$$

$$\frac{I_2 - I_1}{1} + \frac{I_2}{1} + \frac{I_3}{2} + \frac{I_3 - I_1}{1} = 0$$

$$-3I_1 + 3I_2 + 3I_3 \quad -4I_1 + 4I_2 + 3I_3 = 0 \quad \text{— (2)}$$

$$I_3 - I_2 = 2A \quad \text{— (3)}$$

Find I_3:

$$I_2 = -2 + I_3 \qquad \boxed{I_2 = I_3 - 2}$$

$$I_3 = 2 + I_2 \quad \text{— (4)}$$

$$-4I_1 + 4I_2 + 3(2 + I_2) = 0$$

$$-4I_1 + 7I_2 + 6 = 0$$

$$4I_1 - 7I_2 = +6$$

$$I_1 = \frac{76}{17} \qquad I_2 = \frac{58}{17} \qquad I_3 = 2 + \frac{58}{17}$$

$$I_1 = \frac{64}{17} \qquad I_2 = \frac{22}{17} \qquad = \frac{92}{17}$$

$$I_3 = 2 + \frac{22}{17} = 3.29$$

$$P_{2\Omega} = \frac{(3.29)^2}{2}$$

$$= 5.41 \Rightarrow P_{2\Omega} = 9.3 \text{ watts} \quad \text{Ans}$$

Nodel

$$(V_1 - 10) + (V_1 - V_2) + 2(V_1 - V_3) = 0$$

$$4V_1 - V_2 - 2V_3 = 10 \quad \text{—— (1)}$$

$$V_2 - V_1 + V_2 - 2 = 0$$

$$-V_1 + 2V_2 = 2 \quad \text{—— (2)}$$

$$2 + 1[V_3] + 2[V_3 - V_1] = 0$$

$$-2V_1 + 3V_3 = -2 \quad \text{— (3)}$$

(1)x2 $\Rightarrow$ $8V_1 - 2V_2 - 4V_3 = 20$

$$7V_1 - 4V_3 = 22 \quad \Rightarrow \quad 14V_1 - 8V_3 = 44$$

$$-2V_1 + 3V_3 = -2 \quad -14V_1 + 21V_3 = -14$$

$$13V_3 = 30$$

$$V_3 = 30/13$$

$$2V_1 = 3V_3 + 2$$

$$V_1 = 58/13$$

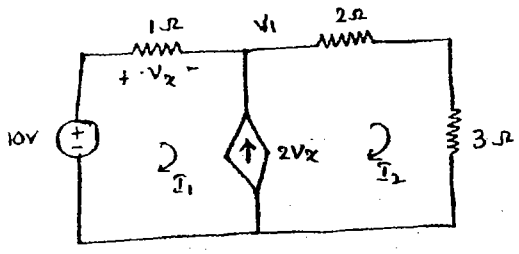
$$V_{drop}(2\Omega) = V_1 - V_3 = 28/13$$

$$P_{2\Omega} = (V_{drop})^2 \cdot G = \left(\frac{28}{13}\right)^2 \times 2 = 9.3 \text{ watts}$$

$P_{2\Omega} = 9.3\text{w}$

WWW.GATENOTES.IN

Problems with dependent Sources:-

① 

Find V_x using mesh & Nodel analysis

Mesh Analysis:-

$$-10 + I_1 + 2I_2 + 3I_2 = 0$$

$$I_1 + 5I_2 = 10 \quad \text{—— ①}$$

$$I_2 - I_1 = 2V_x \quad \text{—— ②}$$

$$I_1 = V_x \quad \text{—— ③}$$

② $\quad 3I_1 - I_2 = 0 \Rightarrow 15I_1 - 5I_2 = 0$

$$I_1 + 5I_2 = 10$$

$$16I_1 = 10$$

$$I_1 = 10/16 = 5/8\ A$$

$$\boxed{V_x = I_1 \cdot 1 = 5/8}$$

Nodel :-

$$\frac{V_1 - 10}{1} + \frac{V_1}{5} - 2V_x = 0$$

$$5V_1 - 50 + V_1 - 10V_x = 0$$

$$6V_1 - 10V_x = 50$$

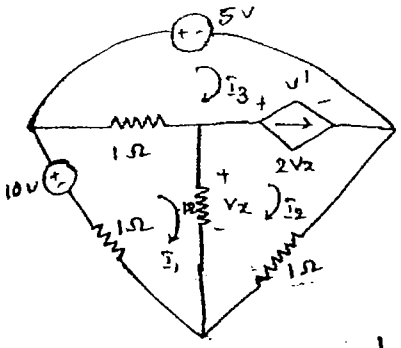
$$3V_1 - 5V_x = 25 \quad \text{— ①}$$

$$V_x = 10 - V_1 \quad \text{— ②}$$

$$V_x = 5/8\ V$$

WWW.GATENOTES.IN

(P)



find the power deliver to by dependent Source by using mesh & nodel.

Mesh

$$I_1 - 10 + (I_1 - I_3) + 1(I_1 - I_2) = 0$$

$$I_2 + (I_2 - I_1) \quad 3I_1 - I_2 - I_3 = 10 \quad \text{—— ①}$$

$$(I_2 - I_1) + (I_3 - I_1) + 5 + I_2 = 0$$

$$-2I_1 + 2I_2 + I_3 = -5 \quad \text{—— ②}$$

$$I_2 - I_3 = 2V_x \quad \text{—— ③}$$

Link Equation : $I_1 - I_2 = V_x \quad \text{—— ④}$

$$I_1 = V_x + I_2$$
$$I_3 = -2V_x + I_2$$

from ①

$$3(V_x + I_2) + I_2 - (-2V_x + I_2) = 10$$

$$-2(V_x + I_2) + 2I_2 + (-2V_x + I_2) = -5$$

$$I_2 + 5V_x = 10$$
$$I_2 - 4V_x = -5 \qquad I_2 = 5/3 \quad V_x = 5/3$$

$V_x = 5/3$

$I_3 = -5/3$ A

$I_2 = 5/3$ A

$I_3 = -5/3$ A

$I_3 = I_2 - 2V_x$

$= \frac{5}{3} - 2(5/3)$

$= -5/3$

$I_1 = 10/3$

$-\frac{5}{3} + v' + \frac{5}{3} = 0$

$v' = 0$

power delivered by dependent Source : $2V_x \cdot (I_2 - I_3)$

$= 2\left(\frac{5}{3}\right)\left(\frac{10}{3} + \frac{5}{3}\right)$

$= 5.55$

$v' \cdot 2V_x$

$= 0$

WWW.GATENOTES.IN

Node 1

$$\frac{V_1 - 10}{1} + \frac{V_1 - V_2}{1} - 2V_x + \frac{V_3}{1} = 0$$

$$2V_1 - V_2 + V_3 - 2V_x = 10 \quad \text{—(1)}$$

$$\frac{V_2 - 4}{1} + \frac{V_2}{1} + 2V_x = 0$$

$$-V_1 + 2V_2 + 2V_x = 0 \quad \text{—(2)}$$

$$V_1 - V_3 = 5 \quad \text{—(3)}$$

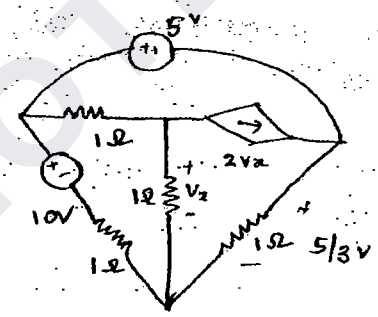
Link $V_x = V_2$ —(4)

By solving $V_x = 5/3$

$V_1 = 20/3$

$V_2 = 5/3$

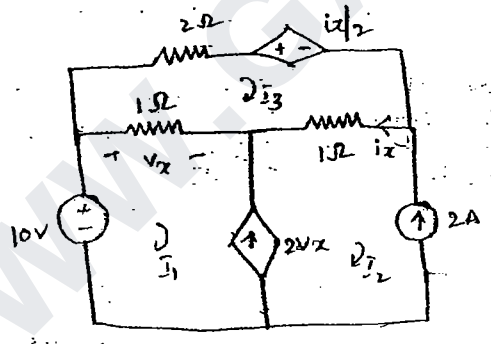
$V_3 = 5/3$



$V_x = 5/3$

power deliver: $2V_x \times V_1 = 2V_x \cdot 0 = 0$

(P)



Find V_x, i_x using mesh & nodal.

Find voltage across all elements

Find Current through all element

Verify KCL at all nodes

Verify KVL at all nodes

Verify Law of Conservation of Energy

WWW.GATENOTES.IN

Meshes.

$-10 + I_1 + (I_3 - I_2)$.

$-(I_1 - I_2) = 2V_x$ ——— (1)

$I_2 = -2A$ ——— (2)

$2I_3 + \frac{i_x}{2} + (I_3 - I_2) + (I_3 - I_1) = 0$

$-I_1 - I_2 + 4I_3 + \frac{i_x}{2} = 0 \Rightarrow -I_1 - I_2 + 4I_3 + \frac{i_x}{2} = 0$

$-2I_1 - 2I_2 + 8I_3 + i_x = 0$ ——— (3)

$V_x = I_1 - I_3$ ——— (4)

$i_x = I_3 - I_2$ ——— (5)

$i_x = I_3 + 2$; $-(I_1 + 2) = 2V_x$

$I_3 = i_x - 2$

$I_1 = -2V_x - 2$ ✓

$I_3 = I_1 - V_x$

$I_3 = -2V_x - V_x - 2$ ✓

$= -3V_x - 2$

from (3)

$-2(-2V_x - 2) - 2(-2) + 8(-3V_x - 2) + i_x = 0$

from (4)

$V_x = I_1 - (i_x - 2)$

$V_x = -2V_x - 2 - i_x + 2$

$3V_x + i_x = 0$

$-20V_x + i_x = 8$

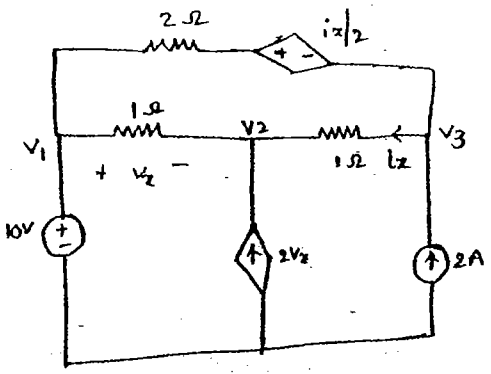
$V_x = -8/23$ $\quad i_x = \frac{24}{23}$

$I_1 = -8/16$ A $\quad I_2 = -2$ A $\quad I_3 = -19/20$ A

$I_1 = -\frac{30}{23}$, $I_2 = -2$ $I_3 = -\frac{22}{23}$ $\quad V_x = -7/20$ $\quad i_x = 21/20$

WWW.GATENOTES.IN

Node 1:



$V_1 = 10$ ——— (1)

$$\frac{V_2 - V_1}{1} + \frac{V_1 - i_x - V_3}{2} = \frac{V_2 - V_1}{1} - 2V_x + \frac{V_2 - V_3}{1} = 0$$

$$-V_1 + 2V_2 - V_3 - 2V_x = 0 \quad —(2)$$

$$-2 + \frac{V_3 - V_2}{1} + \frac{V_3 - V_1 + i_x/2}{2} = 0$$

$$-2V_1 - 4V_2 + 6V_3 + i_x = 8 \quad —(3)$$

$$V_x = V_1 - V_2 \quad —(4)$$

$$i_x = V_3 - V_2 \quad —(5)$$

(2)

$$-10 + 2V_2 - V_3 - 2(V_1 - V_2) = 0$$

$$4V_2 - V_3 = 30 \quad —(6)$$

$$-2(10) - 4V_2 + 6V_3 + V_3 - V_2 = 8$$

$$-5V_2 + 7V_3 = 28$$

$V_2 = 10.34$

$V_3 = 11.39$

$$i_x = V_3 - V_2$$

$$= 11.39 - 10.34$$

$$= 1.05$$

$$V_x = 10 - 10.34$$

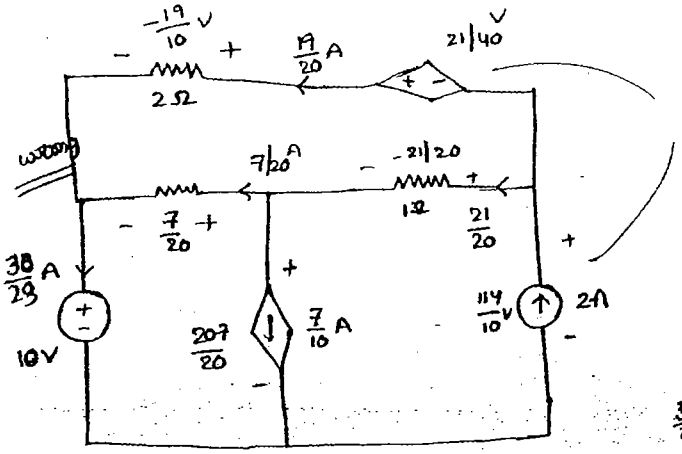
$$= 0.34$$

$V_1 = 10V$

$V_2 = 31/3$ V

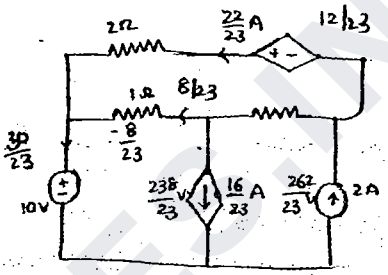
$V_3 = 57/5$ V

WWW.GATENOTES.IN



Sources

Current leaving



Power Sources = $\frac{114}{10}(2) + \left(\frac{21}{40}\right)\left(\frac{19}{20}\right)$

= 23.29

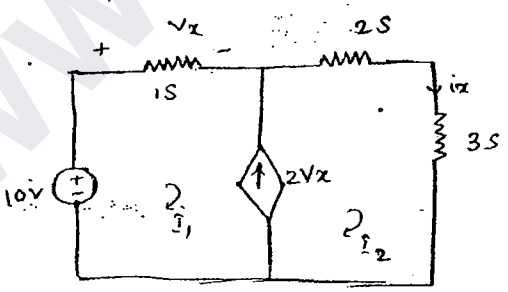
Power Sinks = $10\left(\frac{13}{10}\right) + \frac{207}{20}\left(\frac{7}{10}\right) + \frac{19}{10}\left(\frac{19}{20}\right) + \left(\frac{7}{20}\right)\left(\frac{7}{20}\right) + \frac{21}{20} + \frac{21}{10}$

= 23.27 Watts

Σ psources = Σ psinks.

Law of Conservation of Energy

(P)



Find ix & Vx by using Mesh & Nodal Analysis

$I_2 - I_1 = 2V_x$ — (1)

$-10 + \frac{I_1}{1} + \frac{I_2}{2} + \frac{I_2}{3} = 0$ — (2)

$6I_1 + 5I_2 = 60$ — (2)

$I_2 = i_x$

$V_x = \frac{I_1}{1}$ — (3)

WWW.GATENOTES.IN

$$6I_1 - 2I_2 = 0$$

$$3I_1 - I_2 = 0$$

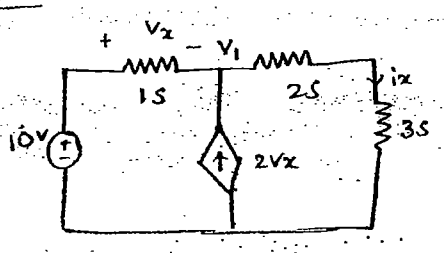
$$6I_1 - 5I_2 = 60$$

$$7I_2 = 60$$

$$I_2 = 60/7$$

$$i_x = I_2 = \frac{60}{7} A$$

Nodal:-



$$(V_1 - 10)1 - 2V_x + V_1\left[\frac{6}{5}\right] = 0$$

$$11V_1 - 10V_x = 50$$

$$V_x = 10 - V_1 \quad \text{— ①}$$

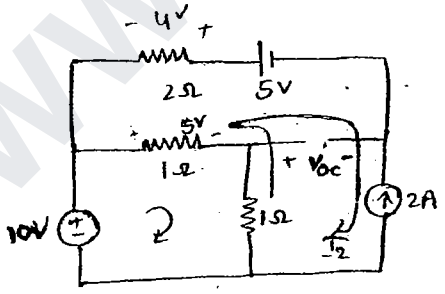
$$11V_1 - 100 + 10V_1 = 50$$

$$21V_1 = 150$$

$$V_1 = \frac{150}{21}$$

$$i_x = V_1\left[\frac{6}{5}\right] = \frac{150}{21} \times \frac{6}{5} = \frac{60}{7} A$$

(P)



find Voc using Mesh & Nodal

$$-10 + (I_1 - I_2)2 = 0$$

$$I_2 = -2A$$

$$I_1 - I_2 = 5$$

$$I_1 = 3A$$

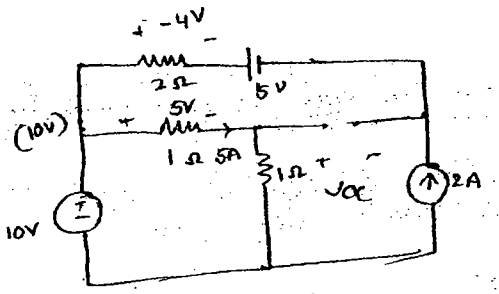
WWW.GATENOTES.IN

KVL

$-4+5-V_{OC}-5=0$

$V_{OC}=-4$

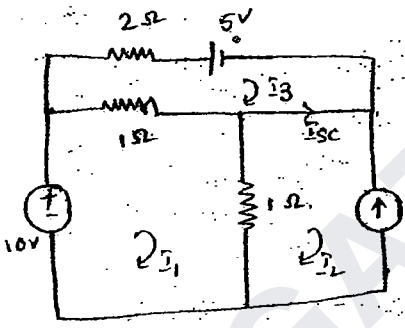
Nodal



$-4+5-V_{OC}-5=0$

$V_{OC}=-4$ volts

(P)



Find I_{SC} using Mesh and Nodal.

7

6-4+3

Mesh

$I_2=-2A$ — ①

$-10V+(I_1-I_3)+(I_1-I_2)=0$

$2I_1-I_2-I_3=10$ — ②

2.

$2I_3+5+1[I_3-I_1]=0$

$-I_1+3I_3=-5$ — ③

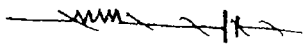
By solving $I_3=-2/5$ $\quad I_{SC}=I_2-I_3$

$=-2+\frac{2}{5}$

$=-8/5$

WWW.GATENOTES.IN

Nodal :-



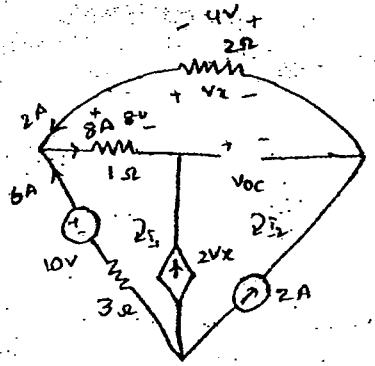
$V_1 = 10$

$$\frac{V_2 - 10}{1} + \frac{V_2}{1} - 2 + \frac{V_2 - 10 + 5}{2} = 0$$

$$5V_2 = 29$$

$$V_2 = 29/5$$

(P)



find open ckt voltage using mesh & nodal

Mesh Analysis:

$$I_2 - I_1 = 2Vx \quad — ①$$

$$I_2 = -2A$$

$$Vx = 2I_2 \quad — ③$$

$$Vx = -4$$

$$-I_1 + I_2 = 4I_2$$

$$I_1 + 3I_2 = 0$$

$$I_1 - 6 = 0$$

$$I_1 = 6A$$

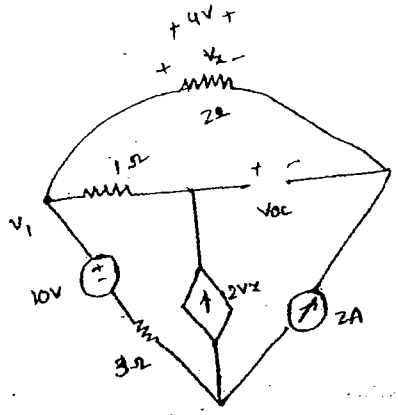
KVL

$$-4 - Voc - 8 = 0$$

$$Voc = 12V$$

WWW.GATENOTES.IN

Nodal



$$\frac{V_1 - 10}{3} - 2V_x - 2 = 0$$

$$V_1 - 10 - 6V_x - 6 = 0$$

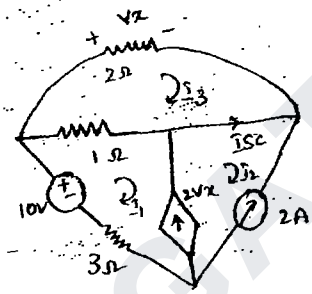
$$V_1 - 6V_x = 16 \quad -(1)$$

$$V_x = -4 \quad -(2)$$

$$V_1 + 24 = 16$$

$$V_1 = -8V$$

(P)



find Isc by using mesh & Nodal.

Mesh Analysis:-

$$I_2 - I_1 = 2V_x \quad -(1)$$

$$I_2 = -2A \quad -(2)$$

$$2I_3 + (I_3 - I_1) = 0 \quad -(3)$$

$$V_x = 2I_3 \quad -(4)$$

$$3I_3 - I_1 = 0$$

$$3\frac{V_x}{2} - I_1 = 0$$

$$2V_x + I_1 = -2$$

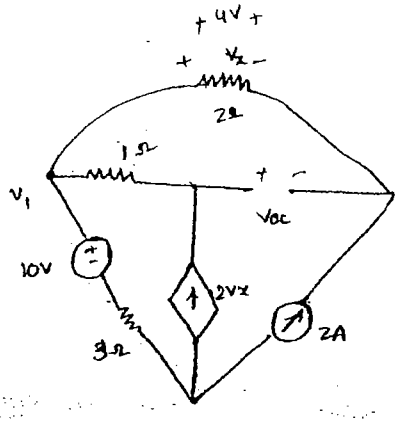
$V_x = -4/7$ $I_1 = -6/7$.

$I_3 = \frac{-4}{14}$

$I_{SC} = -12/7$ A

WWW.GATENOTES.IN

Nodal



$$\frac{V_1 - 10}{3} - 2V_x - 2 = 0$$

$$V_1 - 10 - 6V_x - 6 = 0$$

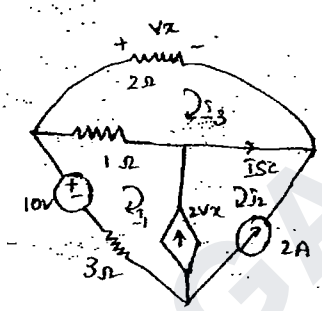
$$V_1 - 6V_x = 16 \quad -(1)$$

$$V_x = -4 \quad -(2)$$

$$V_1 + 24 = 16$$

$$V_1 = -8V$$

(P)



find I_{SC} by using mesh & Nodal.

Mesh Analysis:-

$$I_2 - I_1 = 2V_x \quad -(1)$$

$$I_2 = -2A \quad -(2)$$

$$2I_3 + (I_3 - I_2) = 0 \quad -(3)$$

$$V_x = 2I_3 \quad -(4)$$

$$3I_3 - I_1 = 0$$
$$3\frac{V_x}{2} - I_1 = 0$$
$$2V_x + I_1 = -2$$

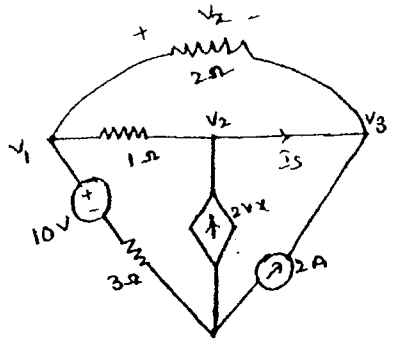
$V_x = -4/7 \quad I_1 = -6/7.$

$$I_3 = \frac{-4}{14}$$

$$I_{SC} = -12/7 \text{ A}$$

WWW.GATENOTES.IN

Nodal



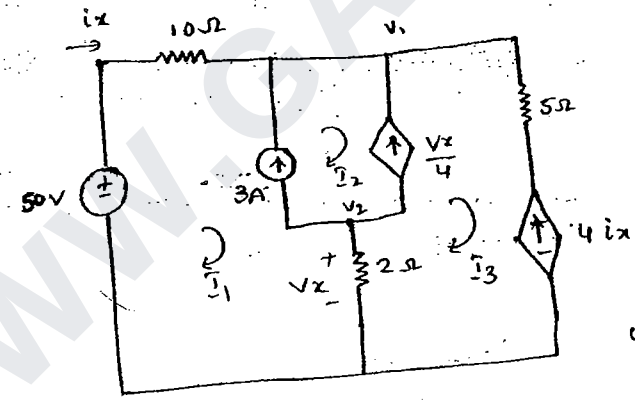
At $\frac{V_1 - V_2}{1} + \frac{V_1 - 10}{3} + \frac{V_1 - V_3}{2} = 0$

$$11V_1 - 6V_2 - 3V_3 = 20$$

$$V_2 = -2V_x$$

Hayl

(P)



write Mesh & Nodal Equations

Mesh analysis:-

Nodes = 5

branches = 7

$m = 3$

$I_2 = -3A$ $\qquad I_2 - I_1 = 3A$

$$-50 + 10I_1 + 5I_3 + 4V_x = 0$$

$$I_3 - I_2 = V_x/4$$

WWW.GATENOTES.IN

$i_x = I_1$

$V_x = 2(I_1 - I_3)$

Nodal :-

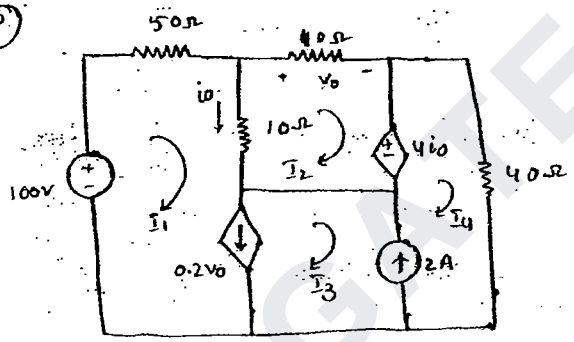
$$\frac{V_1 - 50}{10} - 3 - \frac{V_x}{4} + \frac{V_1 - 4i_x}{5} = 0 \quad -(1)$$

$$\frac{V_2}{2} + 3 + \frac{V_x}{4} = 0 \quad -(2)$$

$$i_x = \frac{50 - V_1}{10} \quad -(3)$$

$$V_x = V_2 \quad -(4)$$

(P)



write Mesh & Nodal Equations

Mesh:

$I_1 - I_3 = 2V_o \quad -(1)$ ✓

$I_2 - I_4 = 4i_o \quad -(2)$ ✗

$-I_3 + I_4 = 2A \quad -(3)$ ✓

$i_o = I_1 - I_2 \quad -(4)$ ✓

$-100 + 50I_1 + 10[I_1 - I_2] - 40 + 40I_4 = 0$ ✓

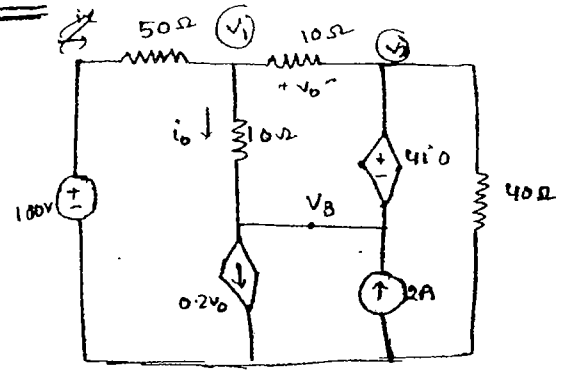
$-100 + 50I_1 + 10I_2 + 40I_4 = 0$

$10[I_2 - I_1) + 10I_2 + 4I_o = 0$

2

WWW.GATENOTES.IN

Nodal



$$\frac{V_1 - 100}{5} + \frac{V_1 - V_3}{10} + \frac{V_1 - V_2}{10} = 0 \quad -(1)$$

$$\frac{V_2 - V_1}{10} + \frac{V_2}{40} - 2 + 0.2v_o + \frac{V_3 - V_1}{10} = 0 \quad -(2)$$

$$V_2 - V_3 = 4i_o \quad -(3)$$

$$v_o = V_1 - V_2 \quad -(4)$$

$$i_o = \frac{V_1 - V_3}{10} \quad -(5)$$

WWW.GATENOTES.IN

Network Theorems:-

Source Transformation Technique

Super position theorem
Thevenins Theorem
Norton's Theorem
} Main

Maximum power Transfer theorm.

IES

Reciprocity theorem

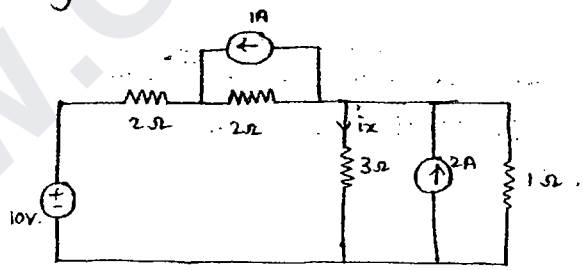
Tellegon's theorem

Milliman's theorem

Compensation theorem

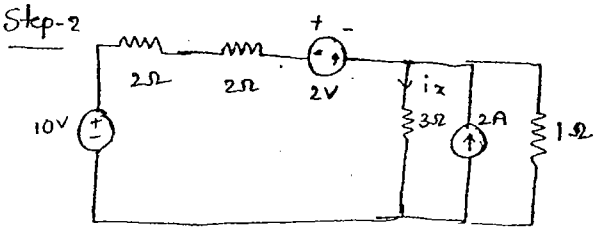
Source Transformation technique:

(1) Find i_x using Source Transformation Technique

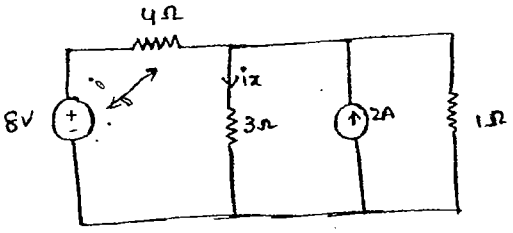


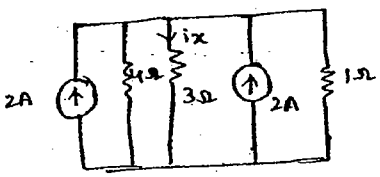
Step-1 don't touch i_x Branch (target)

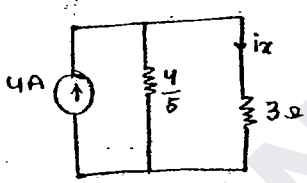
Step-2



WWW.GATENOTES.IN



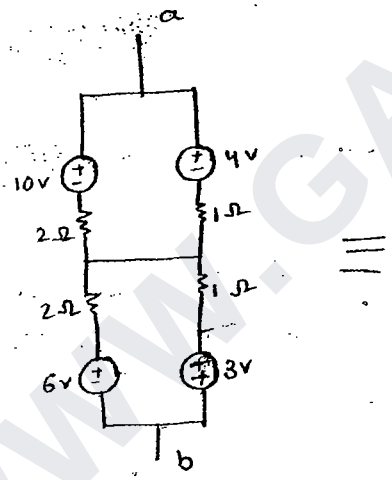




$$i_x = 4\left[\frac{4/5}{\frac{4}{5}+3}\right] = 4\left[\frac{\frac{4}{5}}{\frac{19}{5}}\right] = \frac{16}{19}\ A$$

$$\boxed{i_x = 16/19}$$

(P)



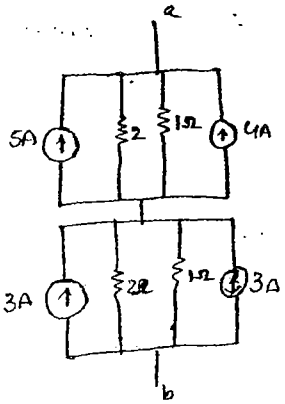
$\equiv$

find

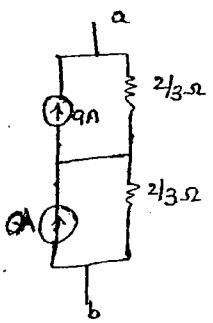
V = ——

R = ——

(A)



$\Rightarrow$



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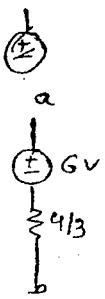


$V = 5R$

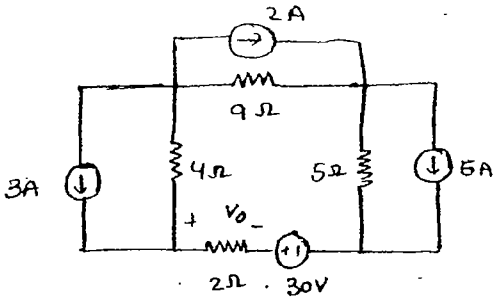
$= 5 \times \frac{2}{3}$

$\boxed{V = 6V}$

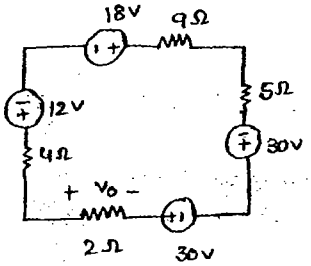
$\boxed{R = 4/3}$

WWW.GATENOTES.IN

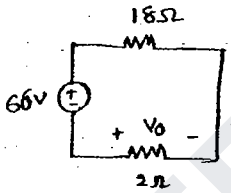
(P)



Find V_o using Source transformation Technique



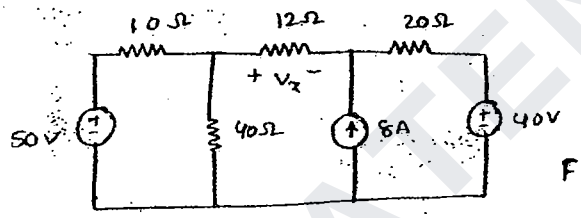
$\Rightarrow$



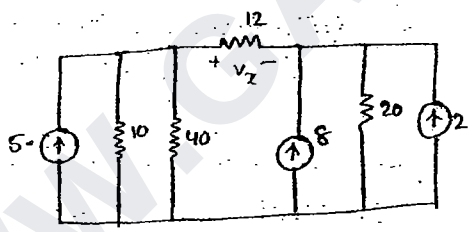
$$V_o = -66\left[\frac{2}{20}\right]$$

$$= -6.6\ V$$

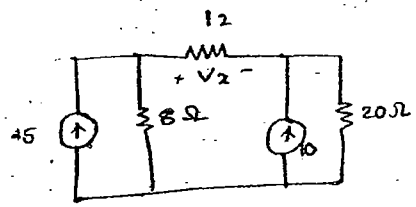
(P)



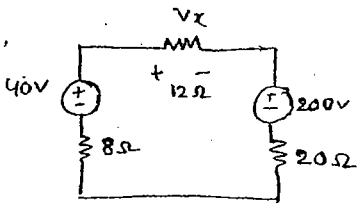
Find V_x using Source transformation



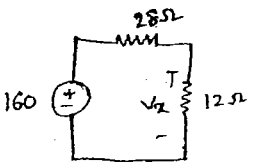
$\Rightarrow$



$\Downarrow$



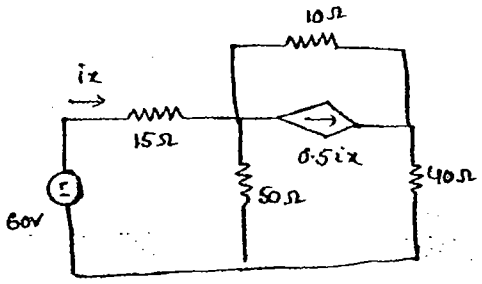
$\Downarrow$



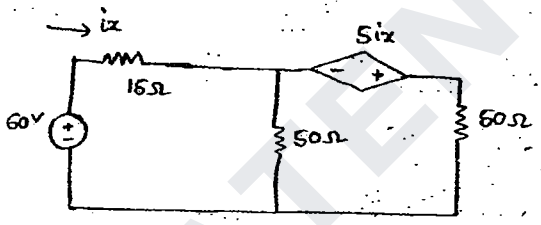
$$-160\left[\frac{12}{40}\right] = -48\ V$$

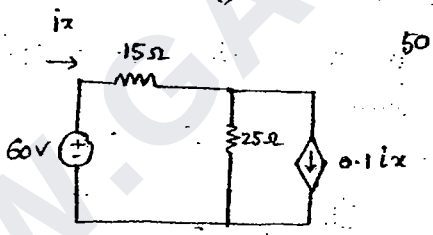
WWW.GATENOTES.IN

(P) Find i_x using Source transformation technique.



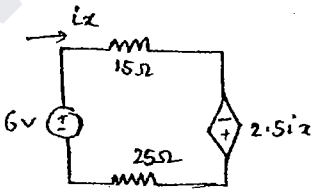
Note: Source transformation technique is also valid for dependent Sources





$50 \| 150 = 25$

$i = \frac{V}{R} = \frac{5i_x}{50} = 0.1 i_x$

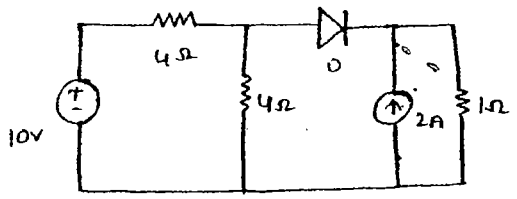


$\Rightarrow -60 + 40 i_x - 2.5 i_x = 0$

$$i_x = \frac{60}{37.5} = 1.6\,A$$

WWW.GATENOTES.IN

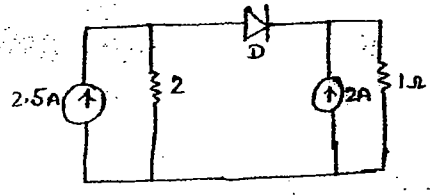
(P)

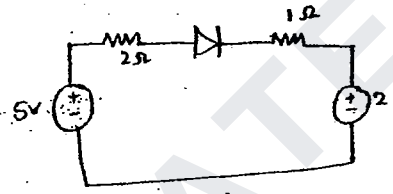


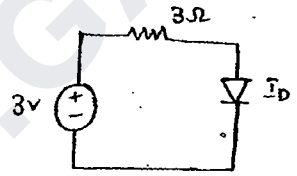
if the diode in the ckt is ideal Find the Current through it

NOTE: Source transformation one of only technique which Can also be applied to Non linear circuits

(4114)







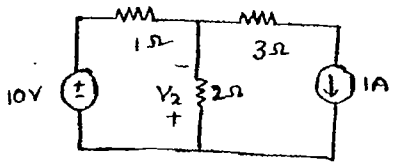
$i_D = \frac{3}{3} = 1A$

Super position theorem:-

..... In any linear active, bilatral Network Consisting of No of Energy Sources & Resistances etc. the effect produced in any element when all Sources atlda time is equal to Sum of Effects in the Same element when each Source is Considered individually.

WWW.GATENOTES.IN

(P)



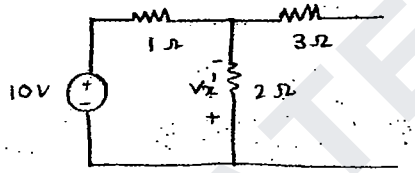
Find V_x using Super position theorem

NOTE:
- Suppressing Ideal Independent Voltage Source means short it
- Suppressing Ideal Independent Current Sourc means open ckt it

Step-1: The No of Sub ckts in Super position theorem
= No of Independent volta Sources.

↓

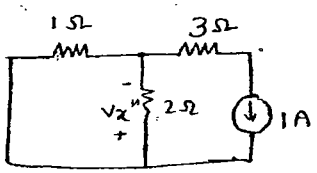
Take 10V



$$V_x' = -10\left[\frac{2}{3}\right]$$

$$= -\frac{10\times 2}{3} = -6.6V$$

Take 1A:



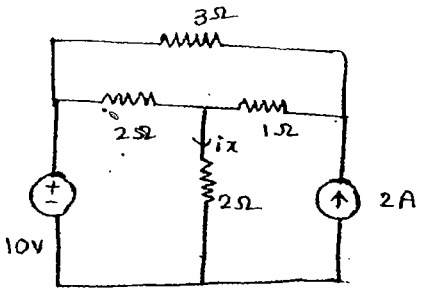
$$V_x'' = 2\left[1\left(\frac{1}{1+2}\right)\right]$$

$$V_x'' = 2/3$$

$$V_x = V_x' + V_x'' = -\frac{20}{3} + \frac{2}{3} = -6V \text{ Ans}$$

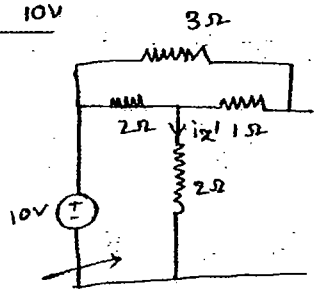
WWW.GATENOTES.IN

(P)



Find ix using Superposition theorem

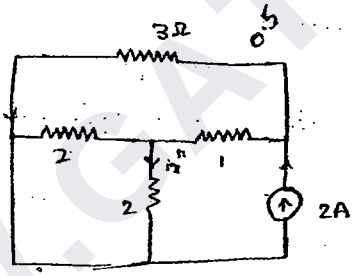
Take 10V



$R_T = 2 + (2 \| 4) = 2 + \frac{8}{6} = \frac{20}{6} = 10/3\ \Omega$

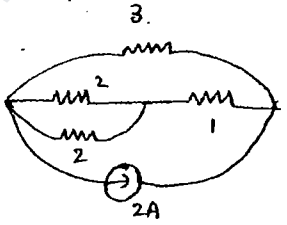
$i_T = \frac{V_T}{R_T} = \frac{10}{10/3} = 3A$

Take 2A:-



$i_x'' = 2\left[\left(\frac{3}{2+3}\right)\left(\frac{2}{2+2}\right)\right]$

$i_x'' = \frac{3}{5}A$



$i_x = i_x' + i_x'' = 3 + \frac{3}{5} = \frac{18}{5}A$

$(2 \| 2 + 1) = 1 + 1 = 2$

2A Shorts 1Ω & 2Ω

$i_x'' = 2\left[\frac{3}{2+3} \cdot \frac{2}{2+2}\right]$

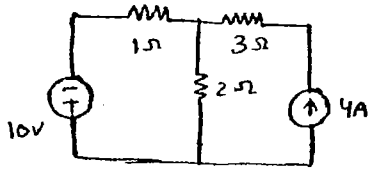
$i_x'' = 3/5$

$i_x = 3 + 3/5 = 18/5\ A$

WWW.GATENOTES.IN

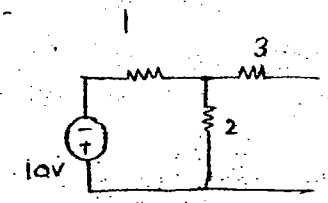
Concept of power in Super position theorem:

(P)



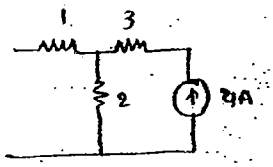
Find power lost in 2Ω Resistance using Super position theorem.

Take -10V :-



$i_{2\Omega} = +10\left[\frac{1}{3}\right] \uparrow$

Take -4A



$i_{2\Omega} = 4\left[\frac{1}{3}\right] = \frac{4}{3} \downarrow$

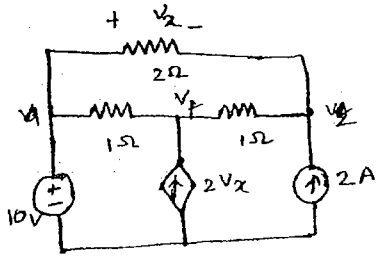
$I_{2\Omega}$ (Resultant) $= \frac{10}{3} - \frac{4}{3} = 6/3 = 2 \uparrow$

power lost $= 2^2 \times 2 = 8$ watts

NOTE :-

Voltages have Specific polarities & Currents have Specific direction it is no advisable to cal Sub power instead find the Resultant voltage or Current & then find power

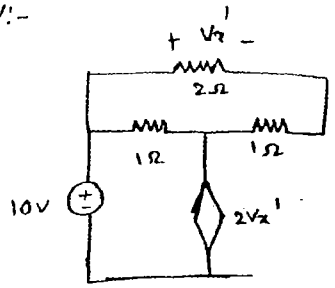
(P)



find V_x using Superposition theorem & Mesh & nodal. Verify by

WWW.GATENOTES.IN

Take 10V:-

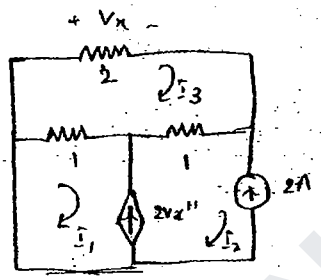


$$V_x' = -2V_x' \left[\frac{1}{1+3}\right]$$

$$V_x' = \frac{-V_x'}{2}$$

$$V_x' = 0$$

Take 2A



$$I_2 - I_1 = 2V_x' \quad -(1)$$

$$I_2 = -2 \quad -(2)$$

$$2I_3 + I_3 - I_2 + I_3 - I_1 = 0$$

$$-I_1 - I_2 + 4I_3 = 0 \quad -(3)$$

$$-2I_1 = 0$$

$$I_1 = 0$$

$$V_x'' = 2I_3 \quad -(4), \quad 2 \times -1/2$$

$$V_x'' = -1$$

$$V_x = V_x' + V_x'' = 0 - 1 = -1V$$

direct verification by nodal:

$$\frac{V_1 - 10}{1} - 2V_x + \frac{V_1 - V_2}{1} = 0$$

$$2V_1 - V_2 - 2V_x = 10 \quad -(1)$$

$$-2 + \frac{V_2 - V_1}{1} + \frac{V_2 - 10}{2} = 0$$

$$-4 + 2V_2 - 2V_1 + V_2 - 10 = 0$$

$$-2V_1 + 3V_2 = 14 \quad -(2)$$

$$V_x = 10 - V_2$$

$$2V_1 - V_2 - 2[10 - V_2] = 10$$

$$2V_1 - V_2 - 20 + 2V_2 = 10$$

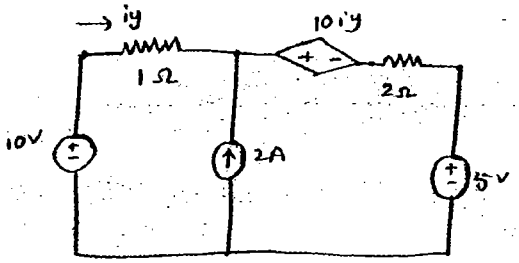
WWW.GATENOTES.IN

$V_2 = 11$

$V_2 = 10 - V_2$

$= 10 - 11 = -1\,V$

(P)



Find iy using Superposition theorm & Verify the Same using mesh/Nodal

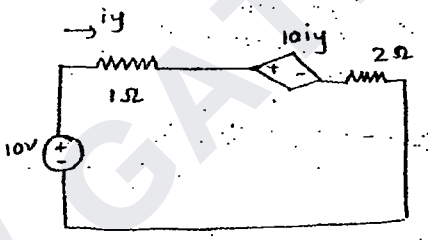
Nodes = 5

Branches = 6

m = 2

Step-1

10V only:



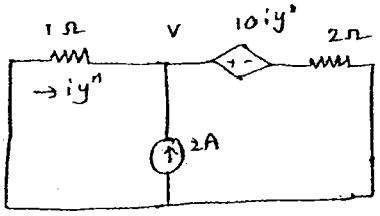
KVL:

$-10 + iy' + 10iy' + 2iy' = 0$

$13iy' = 10 \Rightarrow iy' = \frac{10}{13}$

Independent Source Can't Supress

only 2A



$\frac{V}{1} - 2 + \frac{V - 10iy''}{2} = 0$

$3V - 10iy'' = 2$

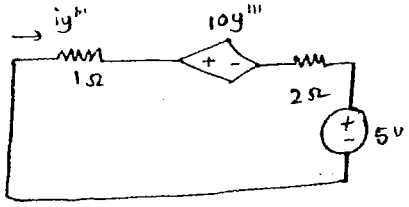
$iy'' = \frac{-V}{1}$

$-13iy'' = 4$

$iy'' = -4/13$

WWW.GATENOTES.IN

Step 3: 5V only



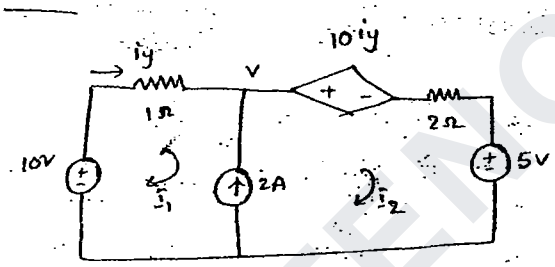
$3i_y'' + 10i_y'' + 5 = 0$

$i_y'' = -5/13$

Final $i_y = i_y' + i_y'' + i_y'''$

$= \frac{10}{13} - \frac{4}{13} - \frac{5}{13} = \frac{1}{13}$ A

direct method:



$-(i_1 - i_2) = 2$

$-10V + i_1 + 10i_y + 2i_2 + 5 = 0$

$i_y = \frac{-10}{1}$

$i_y = -9$

$i_1 = V - 10$

Nodal

$\frac{V-10}{1} - 2 + \frac{V - 10i_y - 5}{2} = 0$

$3V - 10i_y = 29$ — (1)

$i_y = 10 - V$ — (2)

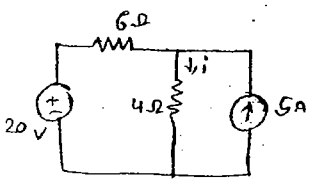
$3[10 - i_y] - 10i_y = 29$

$30 - 13i_y = 29$

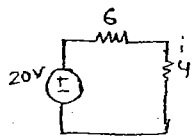
$13i_y = 1 \Rightarrow \boxed{i_y = 1/13}$

IES-06 [3M].

Apply Superposition principle to find i

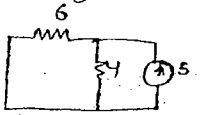


Step-1 only 20V



$i_1 = \frac{20}{10} = 2$

Step-2 only 5A

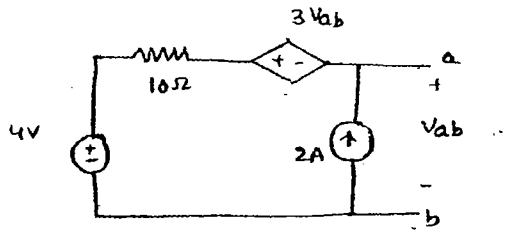


$i_2 = 5\left[\frac{6}{10}\right] = 3$

$i = i_1 + i_2 = 5A$

WWW.GATENOTES.IN

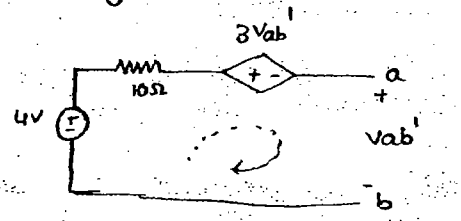
(P) Find Vab using Super position principle



Two independent

So two Sub ckt's

Step 1: only 4V

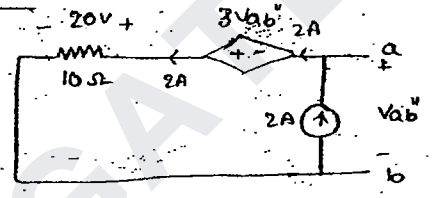


KVL

$$-4 + 3V_{ab}' + V_{ab}' = 0$$

$$V_{ab}' = 1\,V$$

Step 2: only 2A



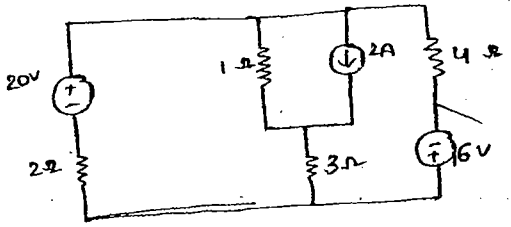
KVL

$$-20 + 3V_{ab}' + V_{ab}'' = 0$$

$$V_{ab}'' = 5$$

Total $V_{ab}'' = V_{ab}' + V_{ab}'' = 1 + 5 = 6V$

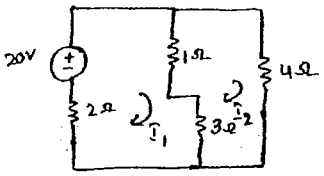
(P) use Super position theorem to Find power deliver to 3Ω Resistor.



Meshes - 3

WWW.GATENOTES.IN

Step-1 only 20V



$2I_1 - 20 + 4I_1 - I_2 = 0$

$6I_1 - I_2 = 20$ — (1)

$4I_2 - 4I_1 + 4I_2 = 0$

$-4I_1 + 8I_2 = 0$ — (2)

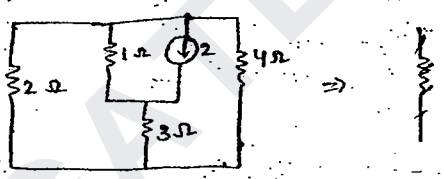
$I_1 = \frac{40}{4}$ $I_2 = 20/4$

$I_1 = 5$ $I_2 = 2.5$

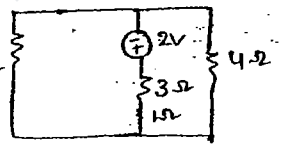
$I_1' = I_1 - I_2 = 5 - 2.5$

$= 2.5A \downarrow$

Step2 only 2A



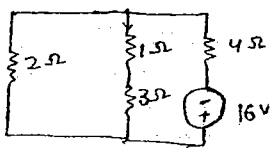
Source Transformation



$R_T = 4 + [2 \| 4] = 16/3\ \Omega$

$i'' = \frac{2}{R_T} = \frac{2}{16/3} = \frac{3}{8} \downarrow$

Step3 only 16V



$R_T = 16/3$

$i_T' = \frac{16}{16/3} = 3A$

$i''' = 3\left[\frac{2}{6}\right] = 1A \uparrow$

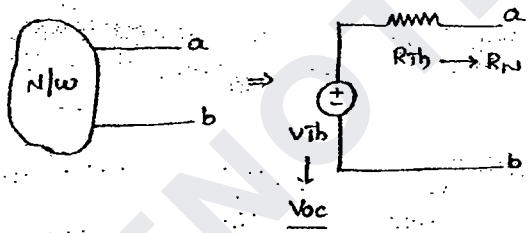
$I = I_1' + I_1'' + I_1''' = 2.5 + \frac{3}{8} - 1 = 15/8\ A$ Power $= \left[\frac{15}{8}\right]^2 3 = 10.546$ Watt

WWW.GATENOTES.IN

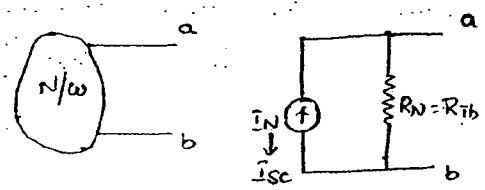
(P) Theorm Number 3 & 4 :-

a) Thevinin's and b) Nortons theorem :-

a) In any linear active bilatral N/w Consisting of No of Energy Sources Resistances etc with open o/p ckt terminals defined Can be Converted in to a Simple ckt Consisting of Voltage Sourca in Series with Resistance



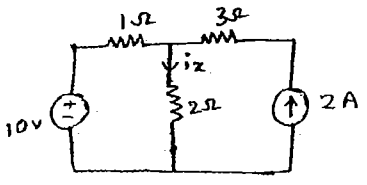
b) In any linear, active, bilatral N/w Consisting of No of Energy Sources Resistances etc with open o/p ckt terminals defined Can be Converted in to a Simple ckt Consisting of a Current Source in parallel with the Resistance.



Category :1 in Nortons & Thevinins theorem:

(problems with independent Sources only)

(P)



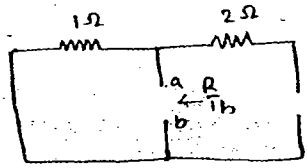
Find iz using thevenins & Nortons theorm

WWW.GATENOTES.IN

→ Remove the target

Step-1 Find R_{Th}

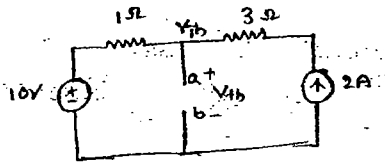
Voltage Source Short Current Source open it



$R_{Th} = 1\Omega$

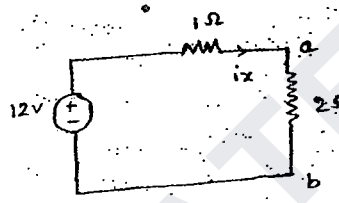
Step-2

Find V_{Th} / V_{OC}



$\frac{V_{Th} - 10}{1} - 2 = 0$

$V_{Th} = 12V$



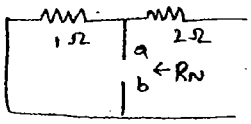
$i_x = 12/3 = 4A$

Norton's theorem:

Step-1 Remove the target

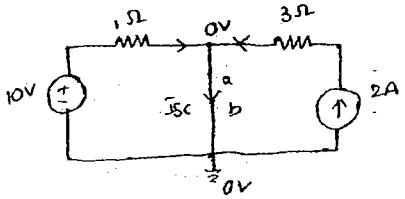
↳ Find R_N.

Voltage Source Short & C.S open



$R_N = 1\Omega$

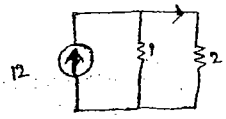
Step2 I_S / I_N



$I_{SC} = \frac{10}{1} + 2$

$= 12A$

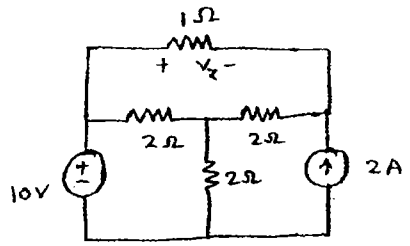
2:1
3



$I_x = 12\left(\frac{1}{3}\right) = 4A$

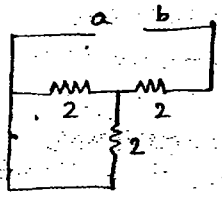
WWW.GATENOTES.IN

(P) Find V_x using thevinins & Nortons



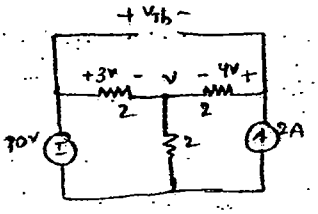
Thevinin's:

Step-1



$R_{Th} = 2+1 = 3\Omega$

Step2



$$\frac{V-10}{2} + \frac{V}{2} + 2 = 0$$

$$2V = 14$$

$$V = 7V$$

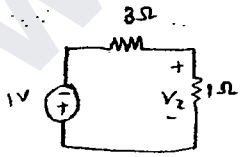
$$V - 10 = V_{2\Omega}$$

$$7 - 10 = V_{2\Omega} = -3$$

$$V_{th} + 4 - 3 = 0$$

$$V_{th} = -1V$$

Step3:

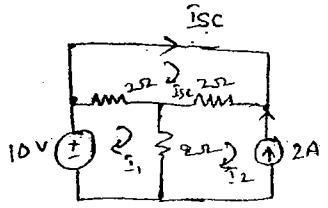


$$V_2 = -1\left[\frac{1}{1+3}\right] = -1/4\ V$$

Nortons theorem:-

$R_N = 3\Omega$

Step2



$$-10 + 2(I_1 - I_3) + 2(I_1 - I_2) = 0$$

$$I_2 = -2A$$

$$2(I_3 - I_2) + 2(I_3 - I_1) = 0$$

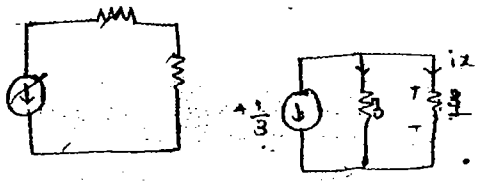
WWW.GATENOTES.IN

$$4\hat{I}_1 - 2\hat{I}_2 - 2\hat{I}_3 = 10$$

$$4\hat{I}_1 - 2\hat{I}_3 = 6$$

$$-2\hat{I}_1 + 4\hat{I}_3 = -4$$

$$\hat{I}_1 = 4/3 \quad \hat{I}_3 = -1/3$$

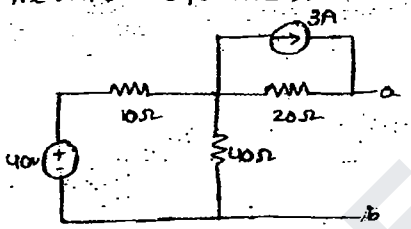


$V_x = \frac{-1}{3}\left[\frac{3}{3+1}\right]1\Omega = -1/4\ V$

(P)

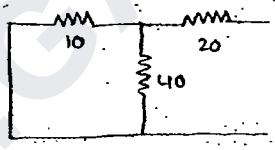
$i_x = \frac{-1}{3}\left[\frac{3}{3+1}\right] = -1/4\ A$

Find thevinis Equivalent at terminal a:b.



Thevinins

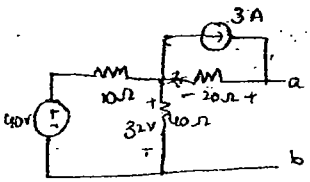
Step-1



$R_{Th} = \frac{10\times 40}{50} + 20$

$= \frac{40}{5} + 20 = 28\Omega$

Step-2



$V_{20\Omega} = +60V$

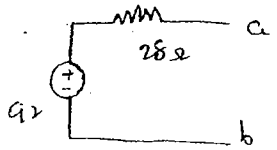
$V_{40\Omega} = 40\left[\frac{40}{50}\right] = 32V$

KVL

$-60V + 32 - V_{Th} = 0$

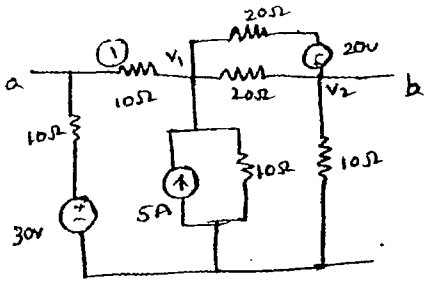
$V_{Th} = 92V$

Step3

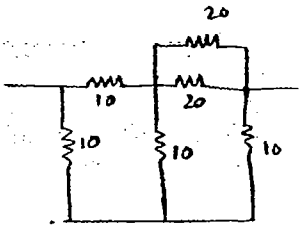


WWW.GATENOTES.IN

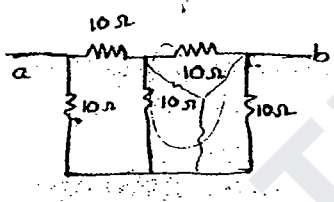
(P) Find Thevinin's equivalent

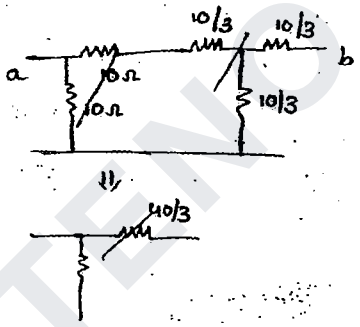


Step-1



⇒





$20 \| 20 = 10\Omega$

Step2

$$\frac{V_1 - 30}{20} - 5 + \frac{V_1}{10} + \frac{V_1 - V_2}{20} + \frac{V_1 - V_2 + 20}{20} = 0$$

$$5V_1 - 2V_2 = 110 \quad \text{—(1)}$$

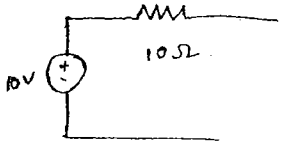
$$\frac{V_2}{10} + \frac{V_2 - V_1}{20} + \frac{V_2 - 20 - V_1}{20} = 0$$

$$-2V_1 + 4V_2 = 20 \qquad V_1 = 30 \quad V_2 = 20$$

Loop

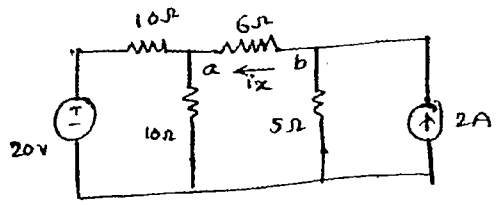
$V_{Th} - 10 + 0 = 0$ $\qquad a - V_{10\Omega} = 0V$ in (1)

$V_{Th} = 10$

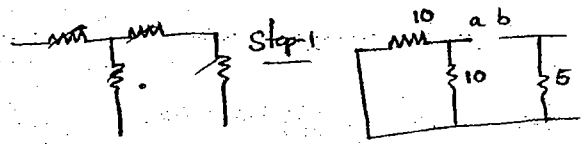


WWW.GATENOTES.IN

(P) Solve i_x using thevinins.



Step-1

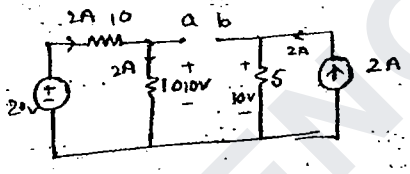


$10 \| 10$

$\frac{10 \times 10}{20}$

V_0 $R_{ab} = 10\Omega$

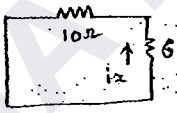
Step2



$10 - 10 + V_{th} = 0$

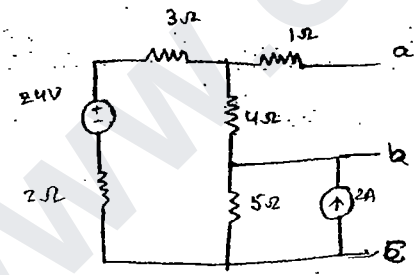
$V_{th} = 0$

Step3



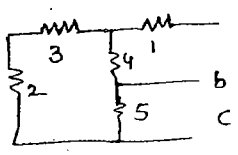
$i_x = 0A$

(P)



Find Nortons & Thevinins Equivalent b/w b & c

Step-1



R_{th}

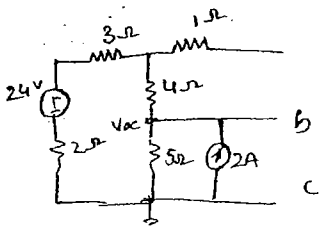
$= 9 \| 5$

$= \frac{45}{14}$

$=$

Use (10V)

$V_{th} =$



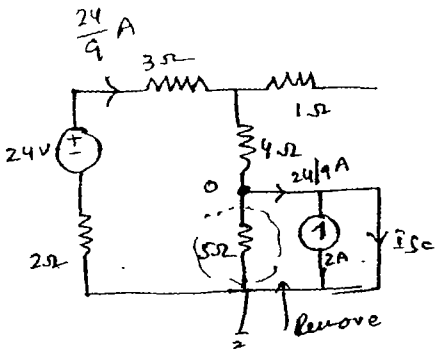
$\frac{V_{oc}}{5} - 2 + \frac{V_{oc} - 24}{9} = 0$

$14 V_{oc} = 210$

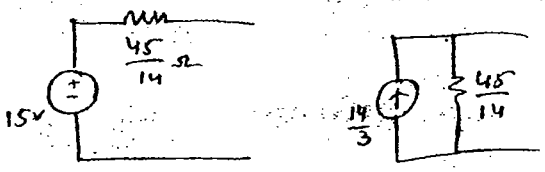
$V_{oc} = 105/7 = 15 \frac{V}{2}$

WWW.GATENOTES.IN

I_{sc}

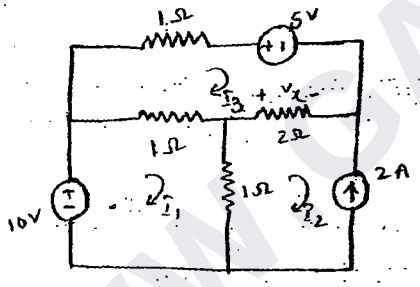


$$I_{sc} = \frac{24}{9} + 2 = \frac{42}{9} = \frac{14}{3}$$



$$R = \frac{V}{I} = \frac{15}{\frac{45}{14}} = \frac{14}{3}$$

(P) Find V_x using Mesh, Nodal, Thevenins, Notons, Superposition



$$-10 + (I_1 - I_3) + (I_1 - I_2) = 0$$

$$2I_1 - I_2 - I_3 = 10 \quad \text{— (1)}$$

$$(I_2 - I_1) + 2(I_2 - I_3) \cdot I_2 = -2A \quad \text{— (2)}$$

$$2(I_3 - I_2) + 1(I_3 - I_1) + I_3 + 5 = 0$$

$$-I_1 - 2I_2 + 4I_3 = -5 \quad \text{— (3)}$$

$$2I_1 - I_3 = 8$$

$$-I_1 + 4I_3 = -9 \qquad I_1 =$$

WWW.GATENOTES.IN

Nodal:

$V_1 = 10 \quad -①$

$$\frac{V_2 - V_1}{1} + \frac{V_2}{1} + \frac{V_2 - V_3}{2} = 0$$

$$-2V_1 + 5V_2 - V_3 = 0$$

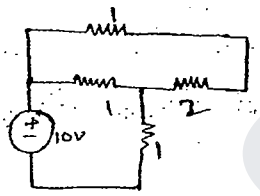
$$\frac{V_3 - V_2}{2} + \frac{V_2 - 4 + 5}{1} - 2 = 0$$

$$-2V_1 - V_2 + 3V_3 = -6 \quad -③$$

$V_x = V_2 - V_3$

Superposition theorem:-

Step-I 10V only



$i_1 = \frac{10}{5/3} = \frac{30}{5} = 6$

$R_T = 1 + (1 \| 3) = 7/4$

$i_T = \frac{V_T}{R_T} = \frac{10}{7/4} = \frac{40}{7}$

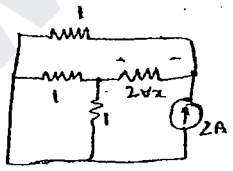
$V_x' = -2\left[\frac{40}{7} \cdot \frac{1}{4}\right] = -\frac{20}{7}$

$\frac{2\times1}{3} + 1$

$5/3$

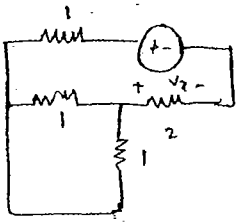
Step2

only 2A only



$V_x'' = -2\left[2\left(\frac{1}{1+5/2}\right)\right] = -4\left[\frac{1}{7/2}\right] = -8/7$

only 3V only



$R_T = 3 + \frac{1}{2} = 7/2$

$i_T = 5/7/2 = \frac{10}{7}$ A

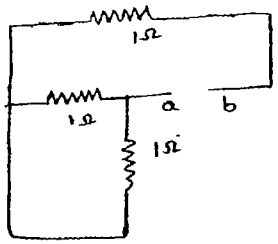
$V_x''' = 2 \times \frac{10}{7} \cdot \frac{2}{7}$

$V_x = V_x' + V_x'' + V_x'''$

$= -8/7$ V

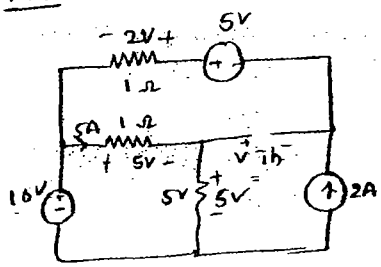
WWW.GATENOTES.IN

Thevenin's theorm:



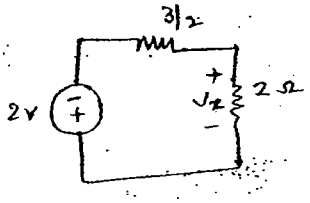
$R_{ab} = R_{Th} = 1 + \frac{3}{2} = 3/2\ \Omega$

V_{th}



$-2 + 5 + -V_{th} - 5 = 0$

$V_{Th} = -2$



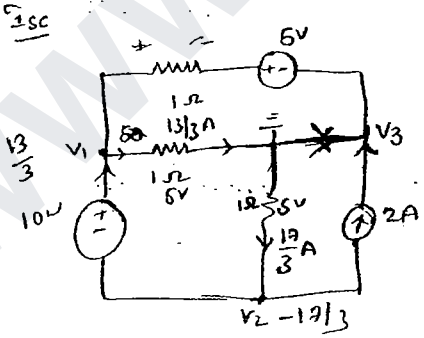
$V_x = -2\left[\frac{2}{2+3/2}\right]$

$= -4/7/2 = -8/7\ V$

Norton's theorm:-

$R_N = 8/2\ \Omega$

I_{SC}



$\frac{V_1 - V_3 - 5}{1} + \frac{V_1}{1} + \frac{V_1 - V_2}{1} + 2 = 0$

$2V_1 + V_2 - V_3 = 0$

$V_1 - V_2 = 10$

$V_3 = 0$

$V_2 = V_1 - 10$

$= \frac{13}{3} - 10$

$V_2 = \frac{-17}{3}$

$2V_1 + V_2 = 3$

$V_1 - V_2 = 10$

$3V_1 = 13$

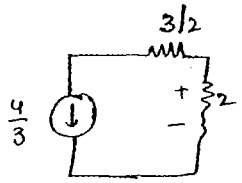
$V_1 = 13/3$

WWW.GATENOTES.IN

KCL

$$\frac{13}{3} = I_{SC} + 17/3$$

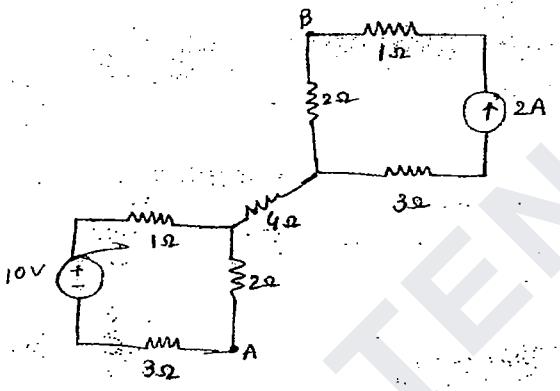
$$I_{SC} = -\frac{4}{3}\ A$$



$R_{eq} = 3/2 + 2 = 7/2$

$$V_{2\Omega} = -\frac{4}{3}\left[\frac{3/2}{3/2+2}\right]2 = -\frac{4}{3}\left[\frac{3}{7}\right]2 = -8/7\ \Omega$$

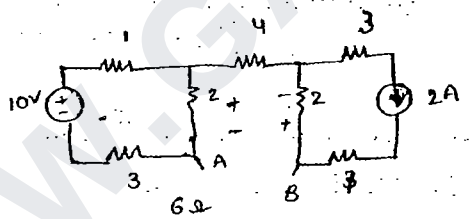
(P)



Thevenins Equivalent across A & B

(Ac)

Thevenins:-



Voltage Shot Current open

10V

$$6 + (2 \| 4) = 6 + \frac{8}{6} = \frac{44}{6}\ (\frac{22}{3})$$

$$V_{2\Omega} \text{ in 10V side} : 10\left[\frac{2}{2+1+3}\right] = 10/3\ V$$

$$I_{2\Omega} \text{ in 2A side} : 2\left[\frac{2}{6}\right] = 2/3$$

$$= 4V$$

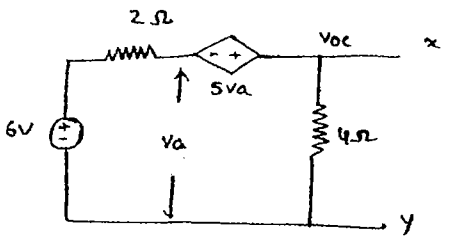
KVL

$$-\frac{10}{3} - 4 - V_{Th} = 0 \Rightarrow -V_{Th} = \frac{10}{3} + 4 : \frac{-22}{3}V$$

WWW.GATENOTES.IN

(P) Find Noton's Equivalent b/w x & y

(problem comes Category - 2 problem)



$R_N = \frac{V_{oc}}{I_{sc}}$ ... $V_{oc} \Rightarrow \frac{V_{oc}}{4} + \frac{V_{oc} - 6 - 5V_a}{2} = 0$

$3V_{oc} - 10V_a = 0$ —(1)

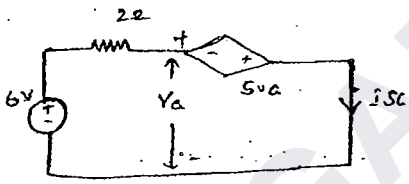
$V_{oc} = 6V_a$ —(2)

$-V_a - 5V_a + V_{oc} = 0$

$3V_{oc} - \frac{10}{6}V_{oc} = 0$

$9V_{oc} - 5V_{oc} = 36 \Rightarrow 4V_{oc} = 36$

$V_{oc} = 9$ volts



$-6 + 2i_{sc} - 5V_a = 0$

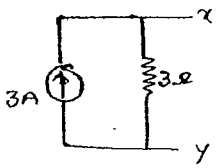
$V_a = 0$

$2i_{sc} = 6$

$i_{sc} = 3$

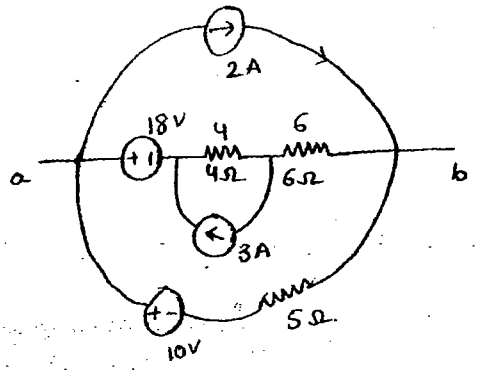
$R_{th} = \frac{V_{oc}}{i_{sc}} = \frac{9}{3} = 3\Omega$

Notons Equivalent

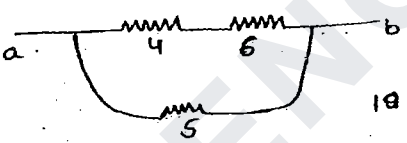


WWW.GATENOTES.IN

For the ckt below Thevenins & Notons equivalent b/w a-b.

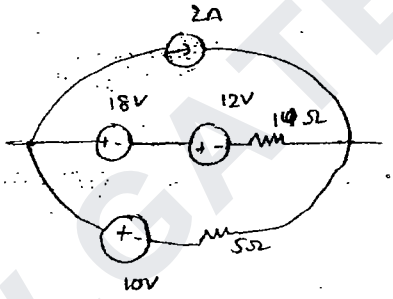


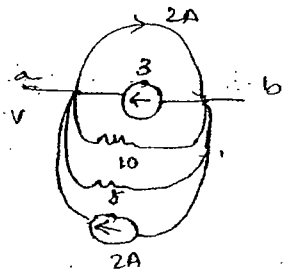
Rab: Voltage Shot, Current Source open



$10 \| 15 = \frac{10 \times 5}{15} = \frac{45}{14} = \frac{50}{15} = \frac{10}{3}\ \Omega$

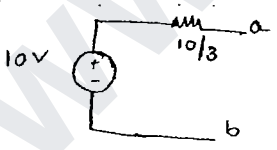
Vth =

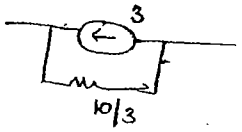




$\frac{80V}{10}$

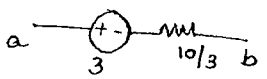
$\frac{10 \| 5}{15}$



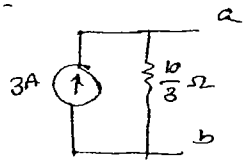


$V = IR$

$3 = 3 \times \frac{10}{3}$



Notons:-

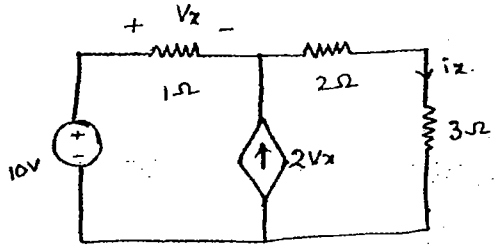


WWW.GATENOTES.IN

Category - 2 problems:

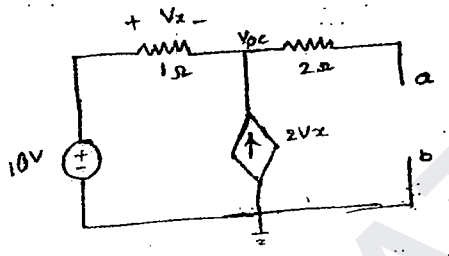
problems with both independent & dependent Sources.

(P)



Find ix using thevenin's & Nortons

$$R_{Th} = R_N = \frac{V_{oc}}{I_{sc}}$$



$$\frac{V_{oc} - 10}{1} - 2V_x = 0$$

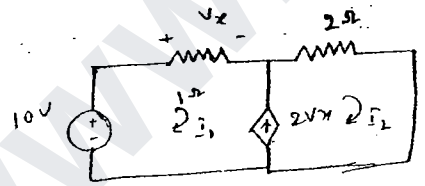
$$V_{oc} - 2V_x = 10$$

$$V_{oc} - 2V_x \qquad V_x = 10 - V_{oc}$$

$$V_{oc} - 20 + 2V_{oc} = 10$$

$$3V_{oc} = 30$$

$$V_{oc} = 10$$



$$-10 + i_1 + 2i_2 = 0$$

$$i_1 + 2i_2 = 10 \quad — (1)$$

$$-i_1 + i_2 = 2V_x \quad — (2)$$

$$V_x = i_1 \quad — (3)$$

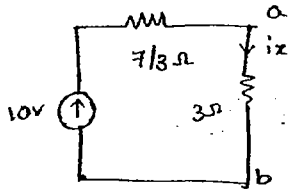
$$i_2 = 30/7$$

$$i_{sc} = 30/7\ A$$

WWW.GATENOTES.IN

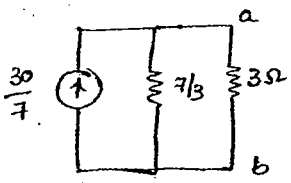
$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{10}{30/7} = 7/3\ \Omega$

$i_x = \frac{10}{3+7/3} = \frac{30}{16} = \frac{15}{8}\ A$



$\therefore i_x = 15/8\ A$

Notons method:

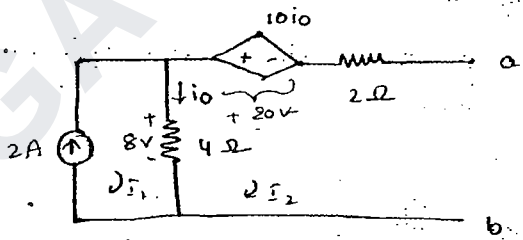


$i_x = \frac{15}{8}\ A$

$i_x = \frac{30}{7}\left(\frac{7/3}{3+7/3}\right) = \frac{15}{8}\ A$

Gate 2008

Determines Norton b/w A & B.



V_{oc}

$-2 + \frac{V_{oc}}{2} = 0$

$V_{oc} = 4$

I_{sc}

$I_1 = 2A$

$4I_2 - 4I_1 + 10i_o + 2I_2 = 0$

$-4I_1 + 6I_2 + 10i_o = 0$

$-4I_1 + 12 + 4i_o = 0$

$-4I_1 + 4i_o = 0$

$4i_o = 4I_1$

$i_o = 2A$

$i_o = I_1 - I_2$

$I_1 - I_2 = i_o$

$2 - I_2 = i_o$

$I_2 + i_o = 2$

$i_o = 2 - 2$

$i_{sc} = 0$

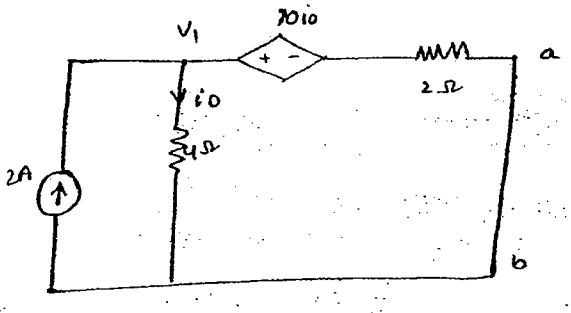
$i_o = 2A$

WWW.GATENOTES.IN

KVL

$-8 + 20 + V_{OC} = 0$

$V_{OC} = -12V$



$-2 + \frac{V_1}{4} + \frac{V_1 - 10i_0}{2} = 0$

$-8. \quad 3V_1 - 20i_0 = 8$

$i_0 = \frac{V_1}{4}$ — (2)

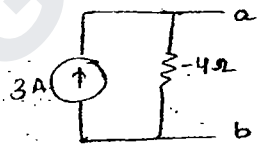
$3V_1 - 5V_1 = 8$

$-2V_1 = 8$

$V_1 = -4 \quad i_0 = -1$

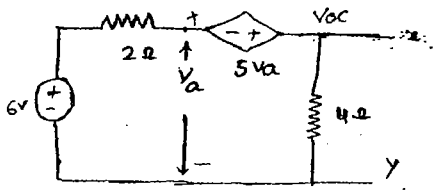
$I_{SC} = \frac{V_1 - 10i_0}{2} = \frac{-4 - 10(-1)}{2} = 3A$

$R_N = \frac{V_{OC}}{I_{SC}} = \frac{-12}{3} = -4\Omega$



gate

Find Nortons Equivalent b/w x & y.



$R_N = \frac{V_{OC}}{I_{SC}}$ $\quad V_{OC} \Rightarrow \frac{V_{OC}}{4} + \frac{V_{OC} - 6 - 5V_a}{2} = 0$

$3V_{OC} - 10V_a = 0$ — (1)

$V_{OC} = 6V_a$ — (2)

$-V_a - 5V_a + V_{OC} = 0$

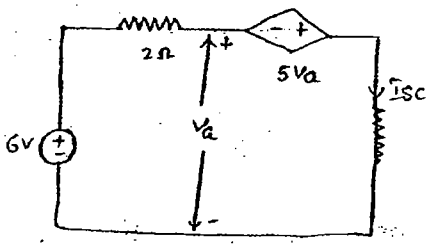
WWW.GATENOTES.IN

$$3V_{oc} - \frac{10}{6} V_{oc} = 0$$

$$9V_{oc} - 5V_{oc} = 36$$

$$4V_{oc} = 36$$

$$V_{oc} = 9 \text{ volts}$$



$$\Rightarrow \quad -6 + 2I_{sc} - 5V_a = 0$$

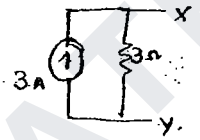
$$V_a = 0$$

$$2I_{sc} = 6$$

$$I_{sc} = 3$$

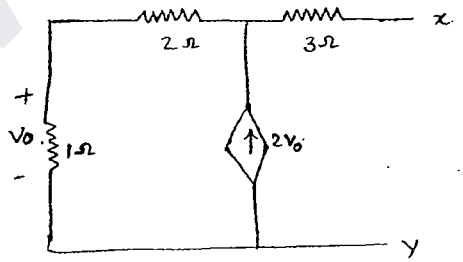
$$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{9}{3} = 3\Omega$$

Nortons Equivalent



Category 3: (problems with only dependent Sources)

(P) Find Thevenins, Nortons Equivalent across xy.

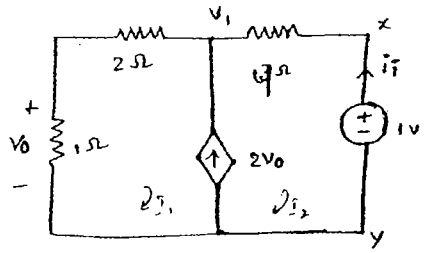


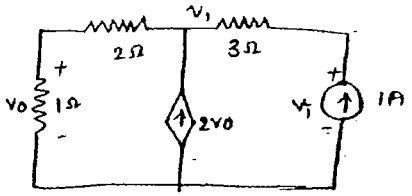
Such models will not have Current Source in Nortons Equivalent & Voltage Source in Thevenins Equivalent. However They will have Resistance

This Resistance Can be determined by Externally Exciting them with a Source

$$R_{Th} = R_N = \frac{1V}{1I} = \frac{V_i}{1A}$$

WWW.GATENOTES.IN





$R_N = R_{Th} = \frac{1}{i_T}$

$R_N = R_{Th} = \frac{V_1}{1}$

$\frac{V_1}{3} - 2V_0 + \frac{V_1 - 1}{3} = 0$ —①

$\frac{V_1}{3} - 2V_0 + V_1 - 1 = 0$

$V_0 = V_1(1/3)$ —②

$V_1 - 6V_0 - 3 = 0$

$V_1 - 6V_0 = 3$

$V_0 = 1\left(\frac{V_1}{3}\right)$

$V_0 = V_1/3$

from ①

$2V_1 - 6V_0 = 1$

$2V_1 - 6\left(\frac{V_1}{3}\right) = 1$

$V_1 = 0$

$i_T = \frac{1-0}{3} = 1/3\ A$

$R_N = R_{Th} = 1/1/3 = 3\Omega$

$7V_1 - 2V_0 = 3$

$7V_1 - 12\left(\frac{V_1}{3}\right) = 3$

$7V_1 - 4V_1 = 3$

$V_1 = 3/3$

$V_1 = 1 = 3$

$V_1 - 6\left(\frac{V_1}{3}\right) = 3$

$-V_1 = 3$

$i_T = \frac{1 - V_1}{4} = \frac{0}{4}$

$= \frac{1 - (-3)}{4}$

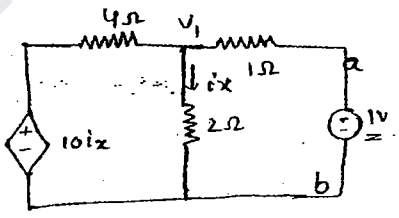
$i_T = \frac{4}{4} = 1A$

$R_N = \frac{-3}{1} = -3$

$i_T = 1$

(P)

obtain Thevenin's Equivalent b/w A & B.



$R_N = R_{Th} = \frac{1}{i_T}$

$\frac{V_1}{2} + 10i_x + \frac{V_1 - 10i_x}{4} + \frac{V_1 - 1}{1} = 0$

$i_x = \frac{V_1}{2}$

$2V_1 + V_1 - 10i_x + 4V_1 - 4 = 0$

$7V_1 - 10i_x = 4$ —①

$7V_1 - 10\frac{V_1}{2} = 4$

$14 - 10V_1 = 8 \quad V_1 = 2//$

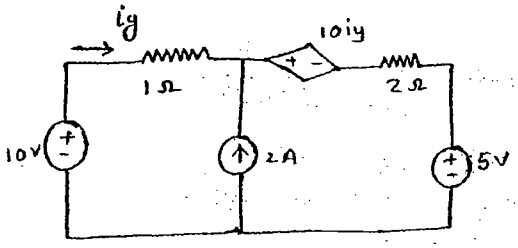
WWW.GATENOTES.IN

$i_1 = \frac{1-2}{4} = \frac{-1}{4}$

$R_{Th} = \frac{-1}{1} = -1\ \Omega$

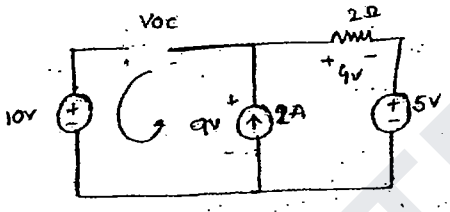
So

Special problems



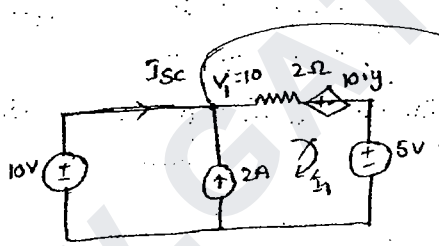
find ig using Thevenins & Notons

in ig 1Ω open ckt so 10 ig = 0 ie Shot ckt



$+10 - 4 - V_{OC} = 0$

$V_{OC} = 1$



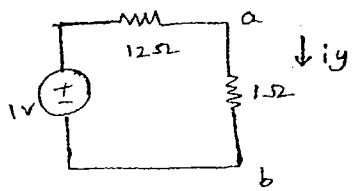
KCL

$2 + I_{SC} = \frac{10 - 5 - 10I_{SC}}{2}$

$4 + 2I_{SC} = 5 - 10I_{SC}$

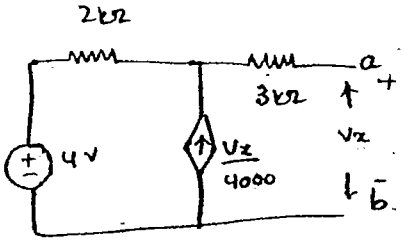
$I_{SC} = 1/12$

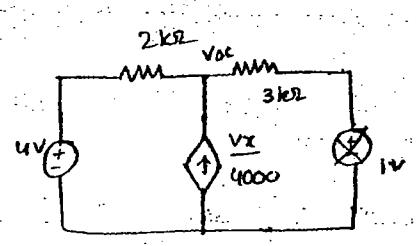
$R_{Th} = \frac{V_{OC}}{I_{SC}} = \frac{1}{1/12} = 12\Omega$



WWW.GATENOTES.IN

Obtain thevenin's b/w a-b





$R_{th} = R_N = \frac{V_{oc}}{I_{sc}}$

$$\frac{V_{oc}-4}{2} - \frac{V_x}{4000} = 0 \quad -①$$

$4 - V_{oc} = 2k\Omega$

$$\frac{V_{oc}-4}{2} + \frac{8}{4000} = 0$$

$$\frac{V_{oc}-4}{2} = \frac{-1}{500}$$

V_{oc}

KVL

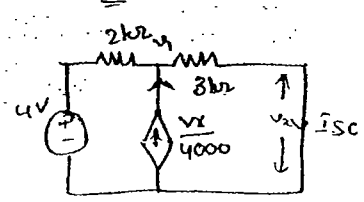
$$+4 + \frac{V_x \cdot 2k\Omega}{4000} = 0$$

$V_x = -8$

$V_{oc} - 4 = \frac{2 \times V_x}{400}$

$V_x =$

I_{sc}



$$\frac{V_{oc}-4}{2000} - \frac{V_x}{4000} = 0 \rightarrow ①$$

$$V_{oc} - 4 = \frac{2V_x 000}{4000}$$

2

$$2V_{oc} - 8 = V_x \quad -②$$

$I_2 - I_1 = \frac{V_x}{4000}$

$-4 + 2k\Omega I_1 = \frac{V_x}{400}$

$$\frac{V_{oc}-4}{2} - \frac{2V_{oc}-8}{4000} = 0$$

$$\frac{V_{oc}}{2} - \frac{2V_{oc}}{4000} - 2 + \frac{1}{500} = 0$$

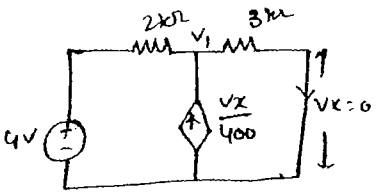
$$\frac{2000V_{oc} - 2V_{oc}}{4000} = 2 - \frac{1}{500} = \frac{999}{500}$$

8

$2000 \quad 1998\, V_{oc} = 8 \times 999$

$$V_{oc} = \frac{8 \times 999}{1998} = 4$$

WWW.GATENOTES.IN



$$-4 + 5I = 0$$

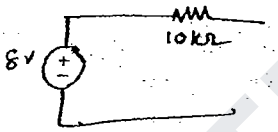
$$I_{SC} = 4/5k$$

$$\frac{V_1 - 4}{2} - \frac{V_x}{4000} + \frac{V_1}{3} = 0$$

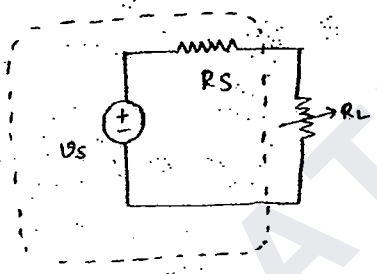
$$R_{Th} = \frac{V_{OC}}{I_{SC}} = \frac{8}{4/5\,k} = 10\,k\Omega$$

$$\frac{V_{OC} - 4}{2} - \frac{V_{OC}}{4k} = 0$$

$$V_{OC} = 8$$



Max^m power Transfer Theorem:- (Can light)



$$I = \frac{V_S}{R_S + R_L}$$

$$P_L = I^2 R_L = \frac{V_S^2}{(R_S + R_L)^2} R_L$$

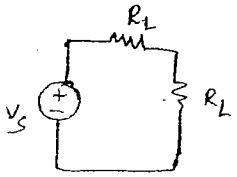
at P_{max}, $\frac{\partial P_L}{\partial R_L} = 0$

$$V_S^2 \left[\frac{(R_S + R_L)^2 \cdot 1 - R_L \cdot 2(R_L + R_S)}{(R_S + R_L)^4} \right] = 0$$

$$R_S^2 + R_L^2 + 2R_S R_L = 2R_L^2 + 2R_S R_L$$

$$R_S^2 = R_L^2$$

$$R_L = R_S$$



$$I = \frac{V_S}{2R_L}$$

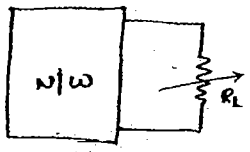
$$P_{max} = I^2 R_L = \frac{V_S^2}{4R_L^2} \cdot R_L$$

$$= V_S^2 / 4R_L$$

WWW.GATENOTES.IN

$$P_{max} = \frac{V_S^2}{4R_L}$$

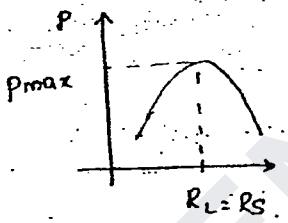
But in general:



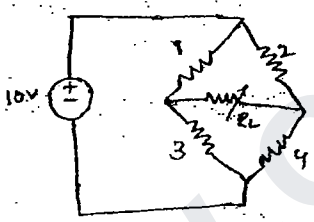
So P_{max} occur when $R_L = R_{Th}$

& this $\left[P_{max} = \frac{V_{Th}^2}{4R_{Th}}\right]$

during max^m power transfer the o/p η = 50%.



(P)

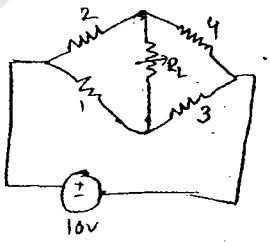


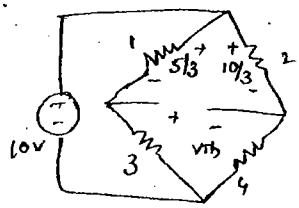
what is max^m power transfer to R_L

R_{Th} = ~~3Ω // 7Ω~~ = $\frac{3\times7}{10}$ = ~~2.1Ω~~ $R_{th} = (1\|3) + (2\|4)$

$= 3/4 + 4/3 = 25/12$

V_{th}





$$-V_{th} - \frac{5}{2} + 2(5/3) = 0$$

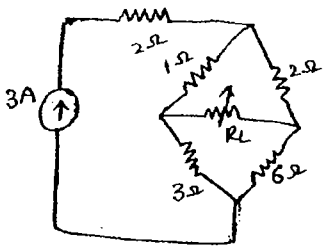
$$V_{th} = \frac{5}{2} - \frac{10}{3}$$

$$V_{th} = -5/6$$

$$P_{max} = \frac{(-5/6)^2}{4(25/12)} = \frac{1}{12} \text{ watt}$$

WWW.GATENOTES.IN

(P)

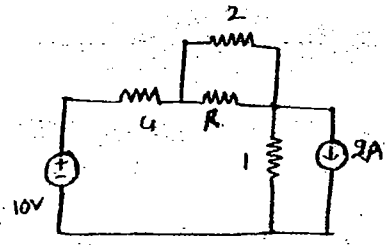


Bridge Balance

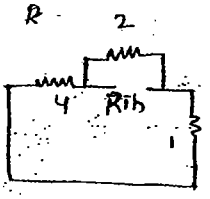
So $I_{R_L} = 0$ ⇒ Current Can't Flow

$P_{max} = 0$

(P)



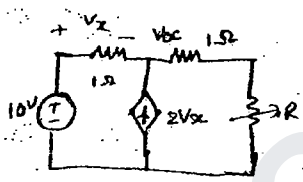
what is the value of R P_{max} occurs in it



$R_{th} = 2 \| 5$

$= \frac{10}{7}\ \Omega$

(P)



what is value of R P_{max} occurs in it

$R_{Th} = \frac{V_{oc}}{I_{SC}}$

$\frac{V_{oc} - 10}{1} + 4 = 2V_x + 0$ V_{oc}

$V_{oc} - 2V_x = 10$ — ①

$V_x = 10 - V_{oc}$ — ②

$V_{oc} - 2(10 - V_{oc}) = 10$

$3V_{oc} = 30$

$V_{oc} = 10V$

I_{SC}

$\frac{V_1 - 10}{1} + \frac{V_1}{1} - 2V_x = 0$

$2V_1 - 2V_x = 10$

$V_1 - V_x = 5$

$V_x = 10 - V_1$

$V_1 - 10 + V_1 = 5$

$V_1 = 15/2$

$R = \frac{V_{oc}}{I_{SC}} = \frac{10}{15/2} = \frac{4}{3}\ \Omega$

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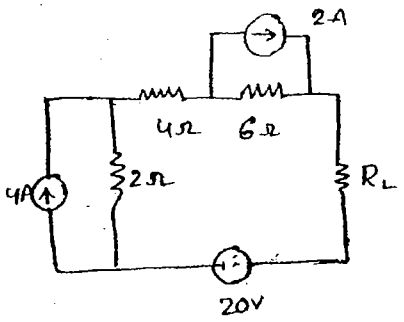
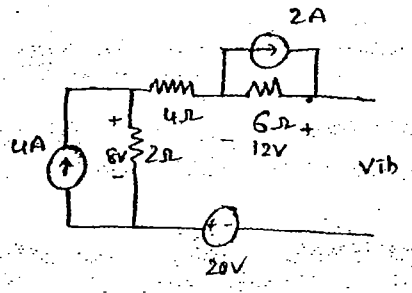
6-1030 PE
11-8 N.W

(P)



Pmax at R_L

$R_{th} = 12\,\Omega$



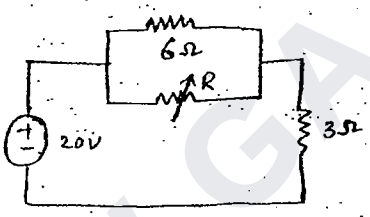
$12 + 8 + 20 = V_{th}$

$V_{th} = 40$

$P_{max} = \frac{(40)^2}{12\,\Omega}$

(P)

Find R ie will give Pmax power to 3Ω Resistor



a) 3Ω
b) 6Ω
c) 9 Ω
d) 12 Ω

Acc to concept

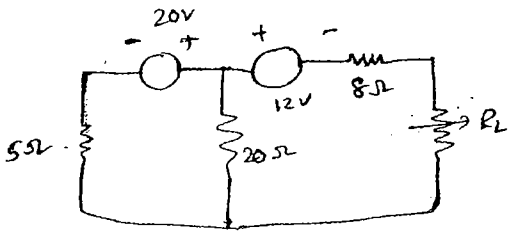
$R_{th} = 6 \| R = 3$

$\frac{6R}{6+R} = 3$

$R = 6\,\Omega$

The least value of R is Correct

(P)



determine Max^m power transfer to R_L

$R_L = 8 + 20 \| 5 = 12\,\Omega$

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$$\frac{V_1 - 20}{5} + \frac{V_1}{20} = 0$$

$$4V_1 - 80 + V_1 = 0$$

$$5V_1 - 80 = 0$$

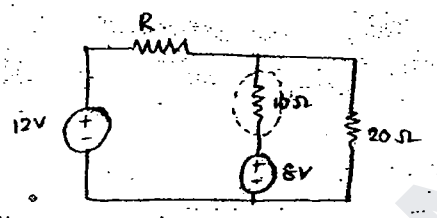
$$V_1 = 16$$

$$-16 + 12 + V_{Th} = 0$$

$$V_{Th} = 4$$

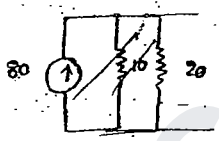
$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{16}{4 \times 12} = 1/3 = 0.33 \text{ watts}$$

(P) Compute the value of R which results max^m power transfer to 10Ω



at 0Ω

$R_{Th} = 10 \| 20 = \frac{200}{30} = 6.66\Omega$



0Ω
5Ω
10Ω
20Ω

$\frac{200}{30}$

$R_{Th} = \frac{20R}{20+R} = 10$

$20R = 200 + 10R$

$R = 20\Omega$ ✗

Reciprocity theorem:-

The Ratio of o/p to i/p Remains the Same in any element Even if the position of Source & Load are inter changed. Reciprocity theorem is not universal theorem.

→ it Can't be applied to multiple Source problem. → It Can't be applied to dependent Source.

→ While writing Reciprocal n/w voltage Source is place in Series to the Branch & Current Source placed parallel to the Branch

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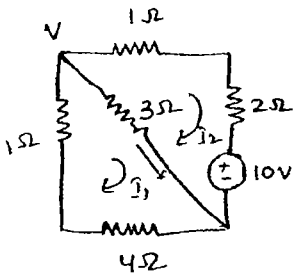
(P)



Find $\underline{I}$ verify Reciprocity principle

$$5\underline{I}_1 + 3(\underline{I}_1 - \underline{I}_2) = 0 \Rightarrow 8\underline{I}_1 - 3\underline{I}_2 = 0 \quad -①$$

$$3(\underline{I}_2 - \underline{I}_1) + 3(\underline{I}_2) - 10 = 0 \Rightarrow -3\underline{I}_1 - 10 = 0$$

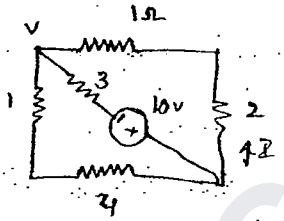
$$-3\underline{I}_1 + 6\underline{I}_2 = 10 \quad -②$$

from ① & ②

$$\underline{I}_1 = \frac{-10}{13} \qquad \underline{I}_2 = -80/39$$

$$\underline{I} = \underline{I}_1 - \underline{I}_2$$

$$= \frac{-10}{13} + \frac{80}{39} = \boxed{50/39}$$



$$\frac{V_1}{5} + \frac{V_1 + 10}{3} + \frac{V_1}{3} = 0$$

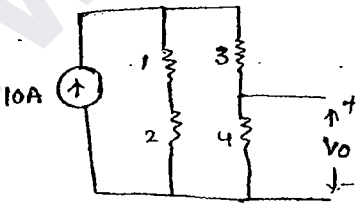
$$3V_1 + 10V_1 + 50 = 0$$

$$13V_1 = 50$$

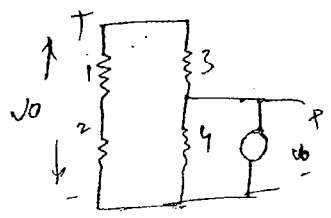
$$V_1 = -50/13$$

$$\underline{I} = \frac{0 + 50/13}{3} = \frac{50}{39}\ A.$$

(P)



Verify Reciprocity principle finding Vo



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(3)(4) $V_0 = 4\left[10\left(\frac{3}{10}\right)\right] = 12V$

from -①

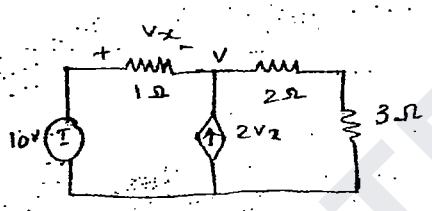
from ② $V_0 = 3\left[10\left(\frac{4}{10}\right)\right]$

$V_0 = 12V$

Tellegen's Theorem:- Verification of Law of Conservation of Energy:

$\Sigma P_{sources} = \Sigma P_{sinks}$.

(P) Verify Tellegen's theorem shown below



$\frac{V-10}{1} + \frac{V}{5} - 2V_x = 0$

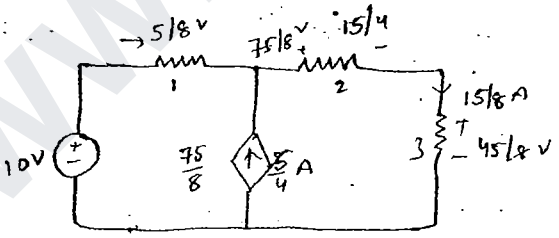
$5V - 10V_x = 50$

$3V - 5V_x = 25$ -①

$V_x = 10 - V$ -②

$V = \frac{75}{8} V$

$V_x = 10 - V = 10 - \frac{75}{8} = \frac{5}{8} V$



$P_{source} = \left[10 \times \frac{5}{8}\right] \times \left[\frac{75}{8} \times \frac{5}{4}\right] = 17.96$.

$P_{sinks} = \left[\frac{5}{8} \times \frac{5}{8}\right] + \left(\frac{15}{4} \times \frac{15}{8}\right) + \frac{45}{8} \times \frac{15}{8} = 17.96V$

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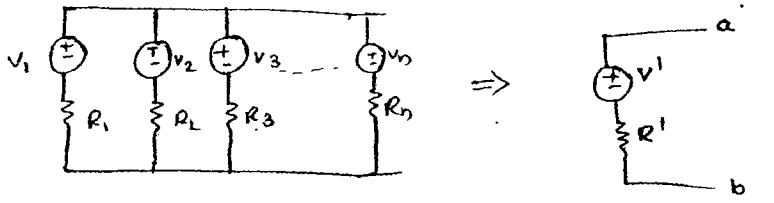
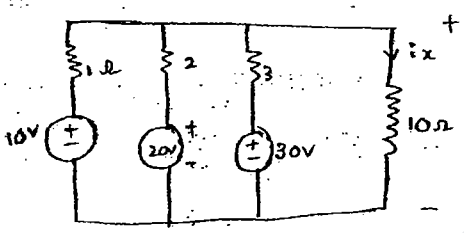
Milliman's theorem & parallel generator theorem



$$V' = \frac{\sum_{i=1}^{n} \frac{V_i}{R_i}}{\sum_{i=1}^{n} \frac{1}{R_i}} = \frac{\sum_{i=1}^{n} V_i G_i}{\sum_{i=1}^{n} G_i} \quad \& \quad R' = \frac{1}{\sum_{i=1}^{n} \frac{1}{R_i}} = \frac{1}{\sum_{i=1}^{n} G_i}$$

(P) Find i_x using Milliman's theorem:

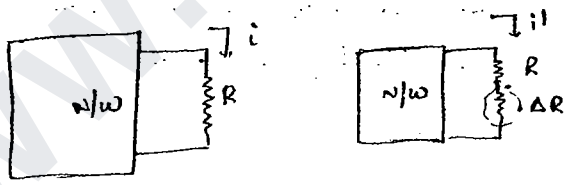


$$V' = \frac{\sum_{i=1}^{n} V_i/R_i}{\sum_{i=1}^{n} \frac{1}{R_i}} = \frac{\frac{10}{1} + \frac{20}{2} + \frac{30}{3}}{1 + \frac{1}{2} + \frac{1}{3}} = 16.3\,V$$

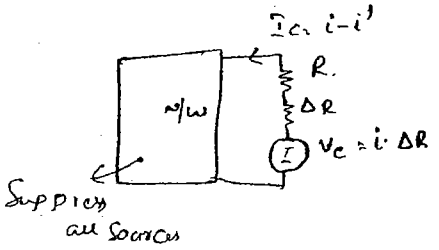
$$R' = \frac{1}{\frac{1}{1} + \frac{1}{2} + \frac{1}{3}} = 0.54\Omega$$

$$i_x = \frac{16.3}{10 + 0.54}$$

Compensation theorem:



Compensation N/W:



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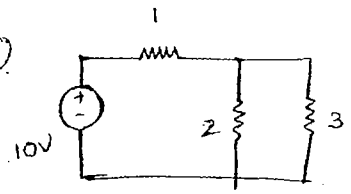
(P)



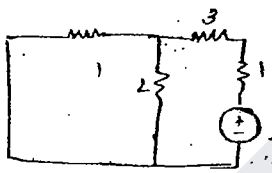
Find the change in Current in 3Ω Resistor branch when an ammeter internal Resistance of 1Ω is measuring Current in it

Step-1:- orginal Current

$$I = 10\left[\frac{1}{1+\frac{6}{5}}\right]\cdot\left[\frac{2}{2+3}\right]$$

$$= \frac{10}{11/5} \times \frac{2}{5} = \frac{20}{11} A$$

Compensated N/w



$$\leftarrow i_C = \frac{\frac{20}{11}}{4+2/3} = \frac{20/11}{14/3} = \frac{30}{77} A.$$

$$\rightarrow V_C = \frac{20}{11} \times 1\Omega = \frac{20}{11}$$

$V_C = i(OR)$

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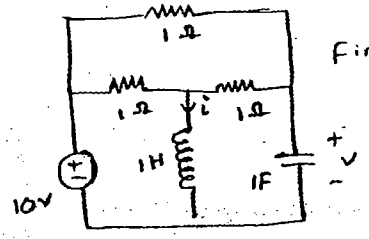
Sinusoidal Steady state Analysis:

our standard Ac Supply is a Sinusoid < Sine / Cosine

(P)



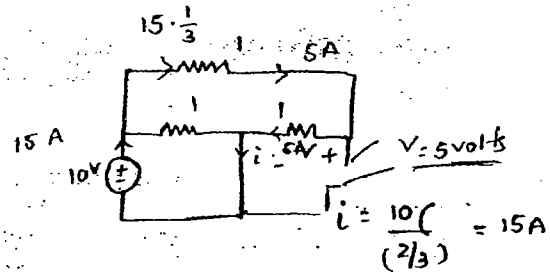
Find i & v in the ckt

DC - Inductor Shot

C - Capacitor open

$V = L\frac{di}{dt} \rightarrow 0 \quad V_L = 0 \rightarrow S.C$

$i = C\frac{dv}{dt} \rightarrow 0 \Rightarrow i_C = 0 \rightarrow o.c$



$i = \frac{10}{(2/3)} = 15A$

In Steady state DC an Inductor acts like Shot ckt & Capacitor acts like open ckt.

properities of Inductor:

$$V = L\frac{di}{dt}$$

→ For DC Inductor acts as Shot ckt $\left[\because \frac{di}{dt} = 0 \Rightarrow V = 0\right.$

→ An Inductor Never allows Current to change Suddenly through it.

$$\uparrow V = L\frac{di}{dt} \rightarrow 0$$

never - to allow Sw during Switch operation ie trasients at that instant Inductor acts open ckt

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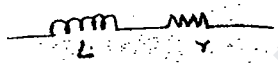
- How Ever when time Varying Sinu Soidal Supply to given to an Inductor it Exhibits Reactance because of it Inductance.
- A pure Inductor is a coil wire whose internal Resistance is Zero.
- practical Inductor How Ever have Resistance which is represented by its Equivalent ckt.



- An Inductor is a passive Element which is used P.S, Communication System. as filters. etc

Properties of Capacitor:-

- Since $i = C\frac{dv}{dt}$
- for DC Capacitor acts as open ckt $\frac{dv}{dt} = 0$, $i = 0$.
- Capacitor never allow Sudden change in Voltage across it (during Transients ie Switch operation Capacitor acts like Short Ckt at that Instant.
- In steady State Sinusoidal analysis it Exhibits Capacitance Reactance because of its Capacitance.
- A pure Capacitor is like parallel charge Conducting plates with infinite dielectric Strength b/w the plates.
- pure Capacitors never Create power loss but practical Capacitor are Consider to have very high Resistance whose Equivalent

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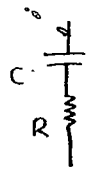
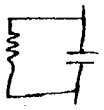
Ckt can be Represented as

Choke coil

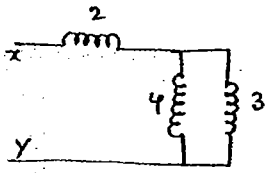




Low voltage

For high voltage

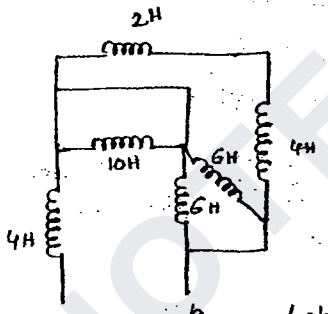
(P)



$L_{xy} = 2 + 4 \| 3$

$= 2 + \frac{12}{7} = \frac{26}{7} H$

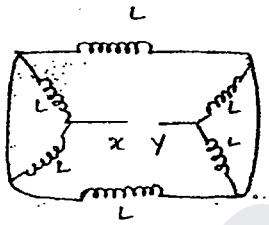
(P)



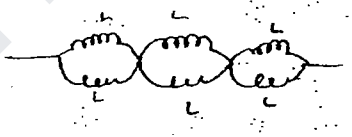
$L_{ab} = 6H$

$\frac{3 \times 6}{7}$ (2)

(P)

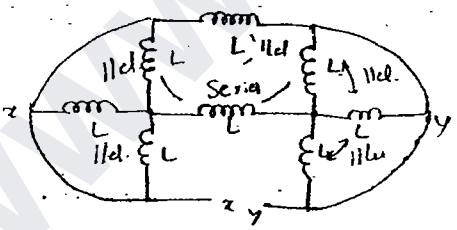


L_{xy} :

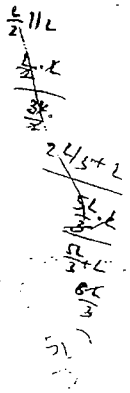


$\frac{L}{2} + \frac{L}{2} + \frac{L}{2} = 3L/2$

(P)



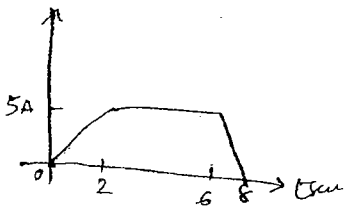
$L_{xy} = \frac{5L}{8}$



(P)

if the Current flowing through 4H Inductor is

then plot the voltage across it



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$$V = L\frac{di}{dt}$$

$$v(t) = L\frac{di(t)}{dt}$$

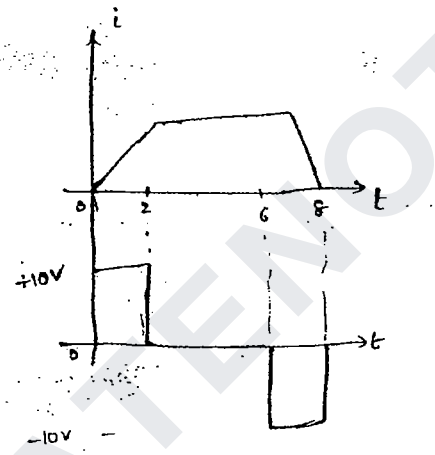
we should determine the Current function : $i(t)$

$$i(t) = \begin{cases} \text{for } 0<t<2 & i(t) = \frac{5}{2}t \Rightarrow v_L(t) = 4\times\frac{5}{2} = 10V \\ \text{for } 2<t<6 & i(t) = 5 \Rightarrow v_L(t) = 4\times 0 = 0 \\ \text{for } 6<t<8 & i(t) = -\frac{5}{2}t + 20 \Rightarrow v_L(t) = 4\times\frac{-5}{2} = -10 \end{cases}$$

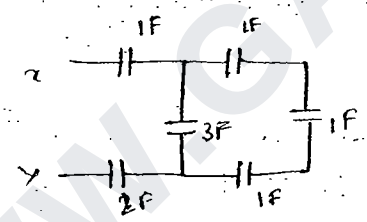
$(6,5)$ $(8,0)$
y_2 y_1

$y - 0 = \frac{5}{2}(x-8)$

$y = -\frac{5}{2}x + 20$



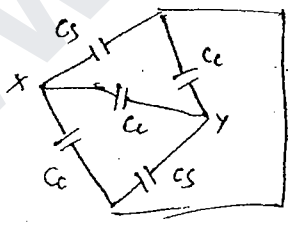
(P)

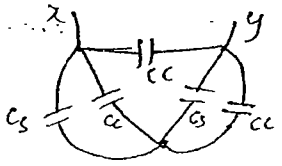


$\frac{1}{1}+\frac{1}{3}+\frac{1}{2} = \frac{6+1+3}{6} = \frac{10}{6} = \frac{5}{3}F$

$(1||1||1) = 1/3 + 3 = (10/3 \;||\; 1||2) = \frac{18}{10} = \frac{9}{5}$

(P)





$= ((C_s + C_c)||(C_s + C_c)) + C_c$

$\frac{3C_c + C_s}{2}$

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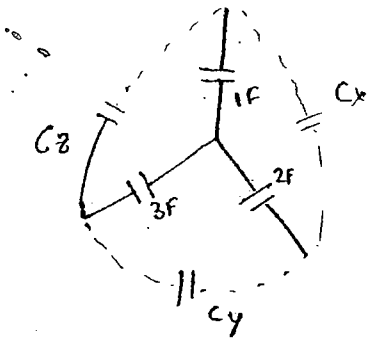
(P)



$$\frac{1}{C_x} = \frac{1}{1} + \frac{1}{2} + \frac{\frac{1}{1}\cdot\frac{1}{2}}{1/3}$$

$$= 1 + \frac{1}{2} + 3/2 = 6/2 = 3$$

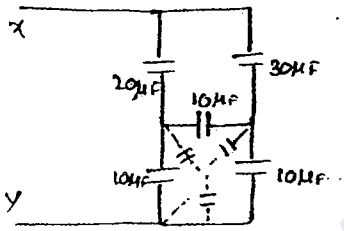
$$C_x = 1/3\ F$$

$$C_y = \frac{1}{2} + \frac{1}{3} + \frac{\frac{1}{2}\cdot\frac{1}{3}}{1/1} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6}$$

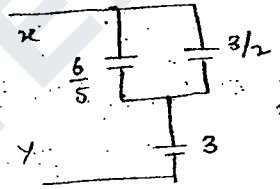
$$\frac{1}{C_y} = 1\ F \qquad \frac{3+2+1}{6} = 1\ F$$

$$C_z = 1/2\ F$$

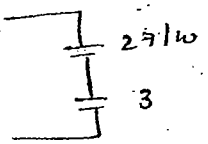
(P)



$\Rightarrow$



$\Rightarrow$



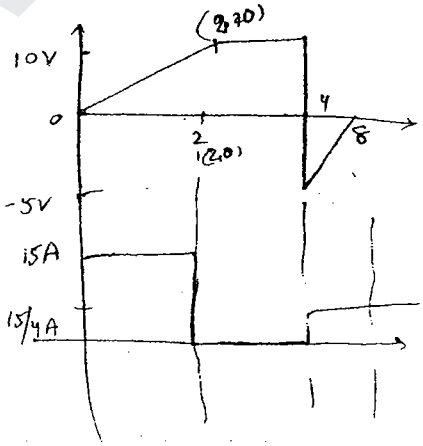
$$C_x = \frac{19}{27}$$

$$\frac{\frac{27}{10}\cdot 3}{\frac{57}{10}} = \frac{27\cdot 3}{57} = 27/19$$

$$C_x = \frac{27}{19} \times 10\,\mu F = 1.42$$

$$= 14.2\,\mu F$$

(P) Voltage across Capacitor 3F Plot Current through it.



$$i = C\frac{dv}{dt}$$

$y - 10 = 5(x-2)$

for $0 < t < 2$

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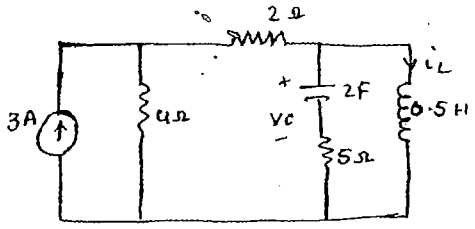
JES-06

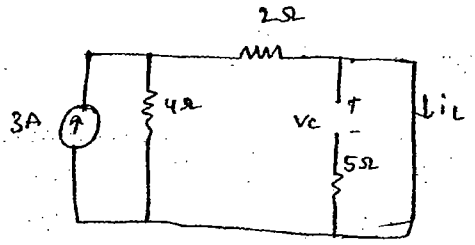
3M



V_c = ——

i_L = ——

(A)



V_c = 0 volts

$i_L = 3\left[\frac{4}{6}\right] = 2A$ 4/3

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Ac Ckt Analysis

(Time invarient Systems).

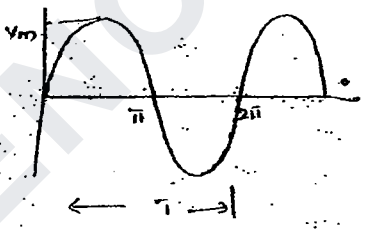
Ac Systems are

- Sine wave
- → Bipolar
- → Bidirectional
- → definite change in phase / polarity
- → power frequency Exist. $f = 50Hz$

→ The standard Ac voltage waveform is Sinusoidal

Radian mode:-

$v = V_m \sin \omega t$



ω - angular frequency (rad/sec)

ωt -

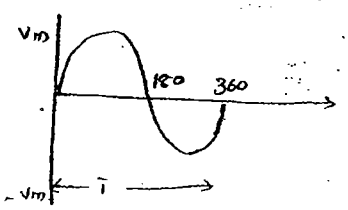
V_m - amplitude

$\omega = 2\pi f = f = \frac{1}{T}$ $\boxed{\omega = \frac{2\pi}{T}}$

1 Time period = 2π radians

degree mode:

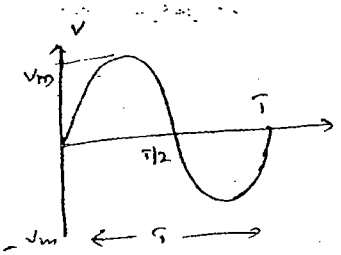
$v = V \sin\theta$



Time period: 360°

Time:

$v = V_m \sin \frac{2\pi}{T} t$



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if phase shift $\theta = 30^\circ$

ie in time $= \frac{30}{360} \times 20\ m\,sec = 1.66\ m\,sec$

$\theta = 90^\circ$

in time $= \frac{90}{360} \times 20\ m\,sec = 5\ m\,sec$

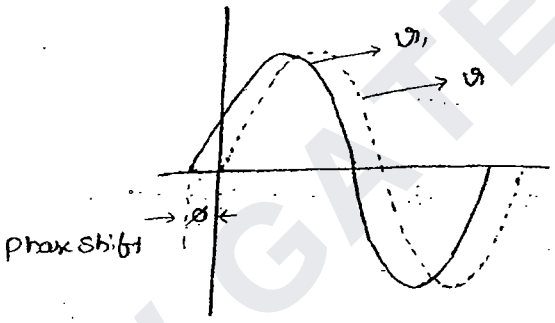
$\Rightarrow$

$\downarrow$

→ Consider, more standard Sinusoidal

$$v_x = V_m \sin(\omega t \pm \phi^\circ)$$

Let $v_1 = V_m \sin(\omega t + \phi)$

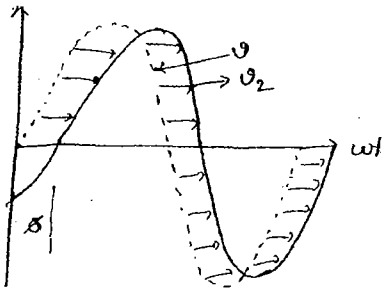


v_1 Comes early than v by ϕ

we Say

v_1 Leads v by ϕ°

Let $v_2 = V_m \sin(\omega t - \phi^\circ)$



v_2 Comes late then v by ϕ°

So we Say

v_2 Lags v by ϕ°

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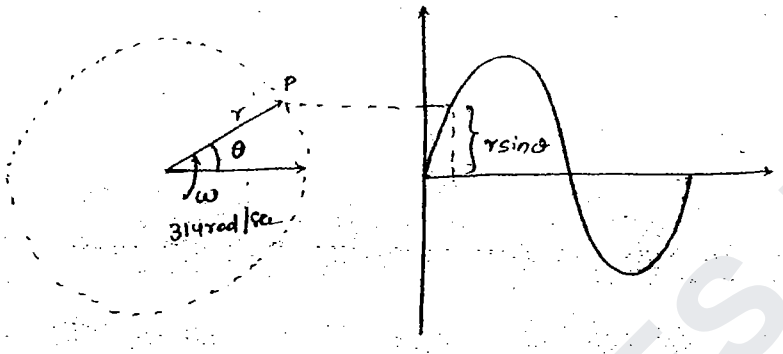
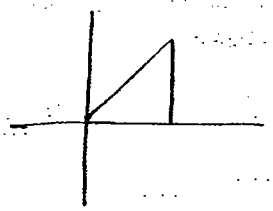
Phasor:-

Time Varying Quantity, phasor rotates in anticlock wise



S-plane



$P = x + jy$ — Rectangular

$P = r\angle\theta$ — polar form

$P = r \cdot e^{j\theta}$ — Exponential form.

Rect – polar $\quad P \rightarrow R$

$r = \sqrt{x^2 + y^2} \qquad x = r\cos\theta$

$\theta = \tan^{-1}(y/x) \qquad y = r\sin\theta$

$R \rightarrow P$

$0 + j0 \rightarrow 0\angle 0$

$1 + j0 \rightarrow 1\angle 0$

$0 + j1 \rightarrow 1\angle 90$ — fast (in time)

$1 + j1 \rightarrow \sqrt{2}\angle 45^\circ$

$0 - j1 \rightarrow 1\angle -90 \rightarrow$ late

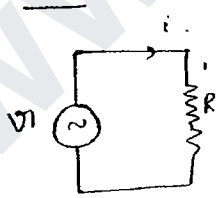
in time Varying Quantity $\rightarrow 1\angle 90^\circ$

$a = \alpha = 1\angle 120^\circ \quad 3\text{-}\phi$

R ——
Y ——— $\uparrow 120^\circ$ phase shift
B ———

phasor Relationship b/w Voltage & Current in parallel passive elements

Resistor:-



Let $v = v_m \sin\omega t$ — ①

$$\bar{I} = \frac{\bar{V}}{R} = \frac{V_m \sin\omega t}{R} = \left[\frac{V_m}{R}\right] \sin\omega t$$

$$I = I_m \sin\omega t \quad \text{— ②}$$

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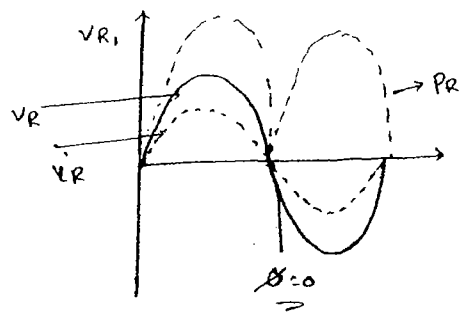
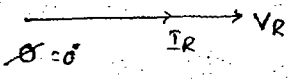


phasor diagram:



→ when a Sinusoidal voltage applied to a resistor same Sinusoidal current flows through it

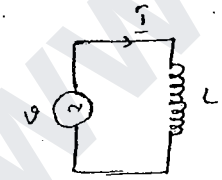
→ Resistor does not involve in any phase shift b/w voltage and current.

→ Resistor is called in phase element. So

$\phi = 0°$, power factor $\cos\phi = \cos 0 = 1$ unity p.f

→ power is always +ve. So resistor dissipates electrical energy in the form of heat. So it is a passive element.

Inductor:-



$$V = L\frac{di}{dt}$$

Let $i = I_m \sin\omega t$

$$V = L\frac{d}{dt}(I_m \sin\omega t)$$

$$V = L I_m \cos\omega t \cdot \omega$$

$$V = \omega L I \sin(90+\theta)^{\omega t + 90}$$

$$V = \omega L I \sin(\omega t + 90)$$

$$V = \omega L I \sin\omega t\ [j]$$

$$V = j\omega L I_m \sin\omega t$$

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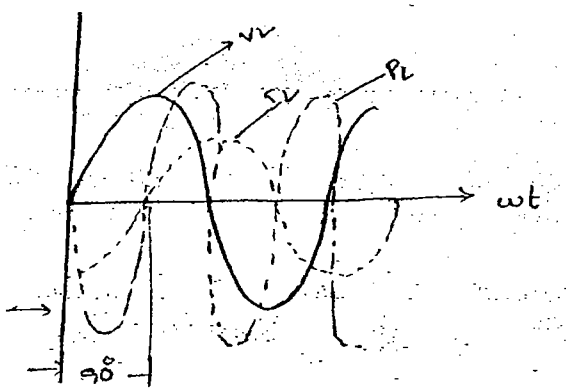
$$\bar{V} = j\omega L \bar{I}$$

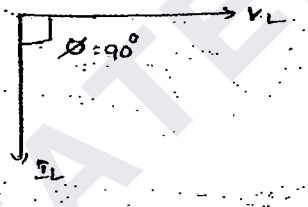
$$\bar{V} = j X_L \bar{I}$$

where $X_L = \omega_L = 2\pi f L$ Inductive reactance Ω



Electromagnetic form

phasor diagram:



V_L leads $\bar{I}_L$ by 90°

When a Sinusoidal voltage given to a Inductor Same Sinusoidal Current flows through it

Inductor involves Creating phaseshift b/w Voltage & Current

In a pure Inductor Current lags Voltage by 90°.

$$\text{So } \theta = 90, \quad \cos\theta = \cos 90^\circ = 0$$

The frequency of Instantaneous power in Inductor is 2 time fundamental frequency

The avg power absorb in a pure Inductor is always zero

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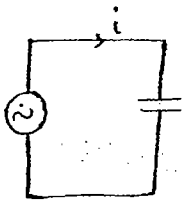
But an inductor stores & releases electrical energy from and back to the Ac supply system in its electro magnetic form.

Capacitor:-



$i = C\frac{dv}{dt}$

Let $v = V_m \sin\omega t$ ——— (1)

$i = C\frac{d}{dt}(V_m \sin\omega t)$

$= C V_m \omega \cos\omega t$

$i = C V_m \omega \sin(\omega t + 90)$

$i = C V_m \omega [J] \sin\omega t$ ——— (2)

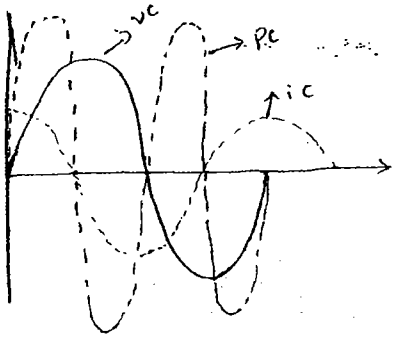
$\bar{I} = j V_m \omega_c \sin\omega t$

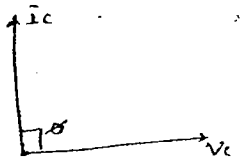
$\bar{I} = j\omega C \bar{V}$

$\bar{V} = \frac{1}{j\omega C}\bar{I} = \frac{-j}{\omega C}\bar{I}$

$V = -jX_C \bar{I}$

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$ Capacitive Reactance in Ω's





$\bar{I}_c$ leads v_c by σ

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when a Sinusoidal voltage given to a Capacitor Same Sinusoidal Current Flows through it

Capacitor involves phaseshift Voltage & Current

Current in a pure Capacitor Leads voltage by 90°

$\theta = 90°$ $\cos 90° = 0$

The instantaneous power frequency in a Capacitor is 2 times fundamental frequency

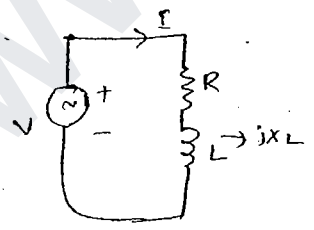
The avg power in pure Capacitor always zero but it continuously Stores & releases Electrical energy from and back to the Ac Supply System in its Electrostatic form

$$\bar{V} = \bar{I}R \quad \& \quad \bar{I} = \frac{\bar{V}}{R}$$

$$\bar{V} = jX_L \bar{I} \quad \& \quad \bar{I} = \frac{\bar{V}}{jX_L}$$

$$\bar{V} = -jX_C \bar{I} \quad \& \quad \bar{I} = \frac{\bar{V}}{-jX_C}$$

Series RL Ckt:



KVL

$$-V + \bar{I}R + jX_L\bar{I} = 0$$

$$\bar{V} = \bar{I}[R + jX_L]$$

$$\bar{V} = \bar{I}Z$$

$$Z = R + jX_L$$

jX_L — Inductive Reactance

R — resistance

Z — impedance

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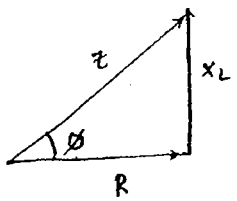
impedance Δle:-



$$|z| = \sqrt{R^2 + X^2}$$

impedance angle ← $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{\omega L}{R}\right)$

$$\cos\phi = \text{p.f} = R/z$$

$$\sin\phi = \frac{X_L}{z}$$

$$\bar{I}^2\bar{z} = \bar{I}^2R + j\bar{I}^2X_L$$

$$V\bar{I} = \bar{I}^2R + j\bar{I}^2X_L$$

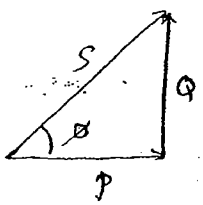
$S = P + jQ_L$ → Reactive power (VAR's)

→ Real power / Active power (watts)

→ Total power / Apparent power (VA's)

Po

power Δle:



$$|S| = \sqrt{P^2 + Q^2}$$

$$\phi = \tan^{-1}\left(\frac{Q}{P}\right)$$

$$\cos\phi = \frac{P}{S} \Rightarrow P = S\cos\phi = V\bar{I}\cos\phi$$

$$\sin\phi = \frac{Q}{S} = Q = S\sin\phi = V\bar{I}\sin\phi$$

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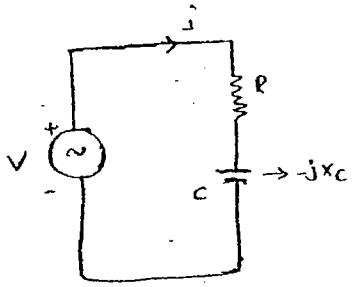
Series RC Ckt:



$$-V + \bar{I}R - jX_C\bar{I} = 0$$

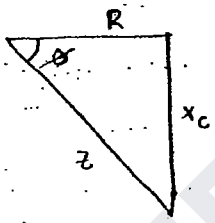
$$V = \bar{I}(R - jX_C)$$

$$V = \bar{I}Z$$

Here $Z = R - jX_C$

- X_C → Capacitive Reactance
- R → Resistance
- Z → Impedance

impedance Δle:-



$$|Z| = \sqrt{R^2 + X_C^2}$$

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$$

$$= \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

$$\cos\phi = p.f = R/Z$$

$$\sin\phi = X_C/Z$$

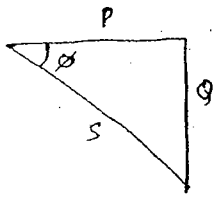
$$\bar{I}^2Z = \bar{I}^2R - j\bar{I}^2X_C$$

$$V\bar{I} = \bar{I}^2R - j\bar{I}^2X_C$$

$$S = P - jQ_C$$

- Q_C → Reactive power (VAR'S)
- P → Real / Active power (watts)
- S → Total power / apparent power (VA'S)

power Δle:-



$$|S| = \sqrt{P^2 + Q^2}$$

$$\phi = \tan^{-1}\left[\frac{Q}{P}\right]$$

$$\cos\phi = \frac{P}{S} \Rightarrow P = S\cos\phi = VI\cos\phi$$

$$\sin\phi = Q/S \Rightarrow Q = S\sin\phi = VI\sin\phi$$

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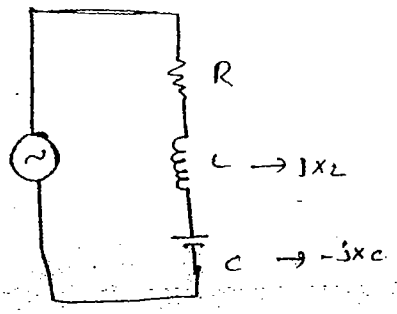
Series R-L-C Ckt



KVL

$-V + \bar{I}R + jX_L\bar{I} - jX_C\bar{I} = 0$

$V = \bar{I}[R + j(X_L - X_C)]$

$V = \bar{I}Z$

Here

$Z = R + j(X_L - X_C)$ ← net Reactance

Case (a)

if $X_L > X_C$ ⇒ Series R-L

($\bar{I}$ lags V, lagging P.f)

Case (b)

if $X_L < X_C$ ⇒ Series R-C

($\bar{I}$ leads V, leading P.f)

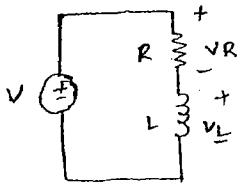
Case (c)

If $X_L = X_C$ ⇒ pure 'R' ckt

($\bar{I}$ in phase with V, UPF) } Concept of resonance

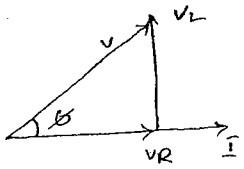
Phasor diagrams:-

1) Series Ckts:- → i is reference



$V_R = \bar{I}R$

$V_L = jX_L\bar{I} = \bar{I}X_L \angle 90^\circ$



$V = \sqrt{V_R^2 + V_L^2}$

$\phi = \tan^{-1}\left[\frac{V_L}{V_R}\right]$

$\cos\phi = \frac{V_R}{V}$ (lagging)

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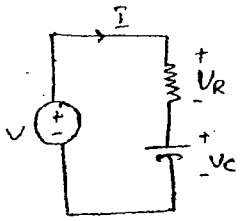
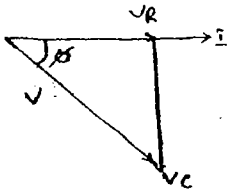
Series R-C ckt:-



$V_R = IR$

$V_C = -jX_C I = I X_C \angle -90^\circ$

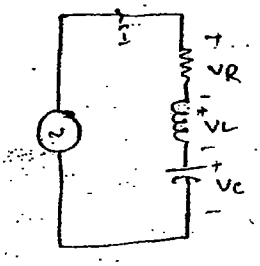


$V = \sqrt{V_R^2 + V_C^2}$

$\phi = \tan^{-1}\left(\frac{V_C}{V_R}\right)$

$\cos\phi = \frac{V_R}{V}$ (leading)

Series R-L-C:

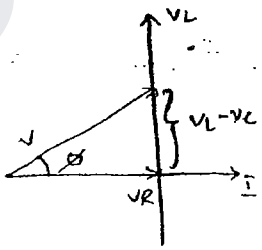


$V_R = IR$

$V_L = IX_L \angle 90$

$V_C = IX_C \angle -90$

Case(i): $X_L > X_C \Rightarrow V_L > V_C$



$V = \sqrt{V_R^2 + (V_L - V_C)^2}$

$\phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right)$

$\cos\phi = \frac{V_R}{V}$ (lagging)

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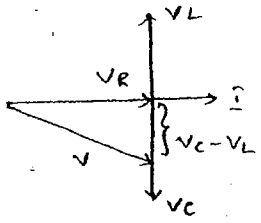
Case ii:

$X_L < X_C \Rightarrow V_L < V_C$

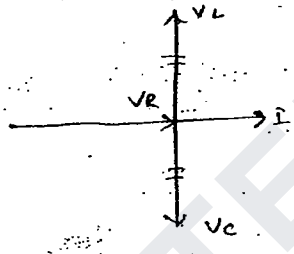


$V = \sqrt{V_R^2 + (V_C - V_L)^2}$

$\phi = \tan^{-1}\left[\frac{V_C - V_L}{V_R}\right]$

$\cos\phi = \frac{V_R}{V}$ (Leading)

Case iii) $X_L = X_C \Rightarrow V_L = V_C$



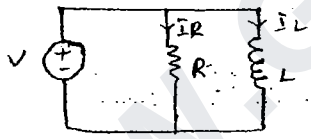
$V = V_R$

$\phi = 0$

P.f: $\cos\phi = 1$ upf

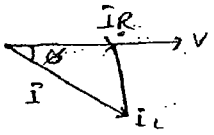
ii) Parallel ckts:

V reference



$\bar{I}_R = \frac{V}{R}$

$\bar{I}_L = \frac{V}{jX_L} = \frac{V}{X_L}\angle -90$

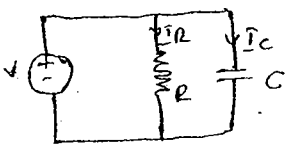


$I = \sqrt{I_R^2 + I_L^2}$

$\phi = \tan^{-1}\left(\frac{I_L}{I_R}\right)$

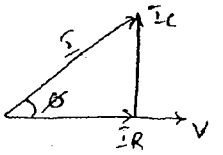
$\cos\phi = \frac{I_R}{I}$ (Lagging)

Parallel R-C ckt:-



$\bar{I}_R = V/R$

$\bar{I}_C = \frac{V}{-jX_C} = \frac{V}{X_C}\angle 90^\circ$



$I = \sqrt{I_R^2 + I_C^2}$

$\phi = \tan^{-1}(I_C/I_R)$

$\cos\phi = I_R/I$ (Leading)

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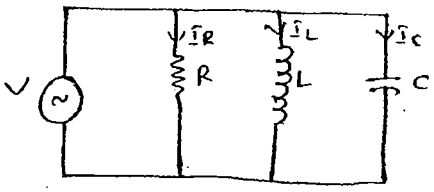
parallel R-L-C Ckt

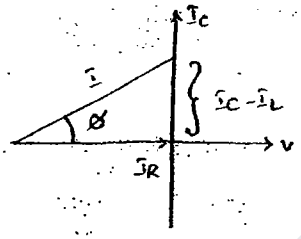


$$\bar{I}_R = \frac{V}{R}$$

$$\bar{I}_L = \frac{V}{jX_L} = \frac{V}{X_L}\angle -90^\circ$$

$$\bar{I}_C = \frac{V}{-jX_C} = \frac{V}{X_C}\angle 90^\circ$$

Case i: $X_L > X_C \Rightarrow I_L < I_C$

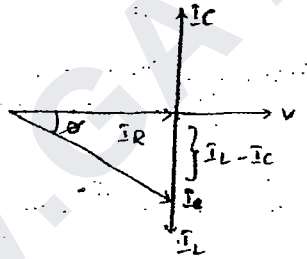


$$I = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$\phi = \tan^{-1}\left(\frac{I_C - I_L}{I_R}\right)$$

$$\cos\phi = \frac{I_R}{I} \text{ (Leading)}$$

Case ii: $X_L < X_C \Rightarrow I_L > I_C$



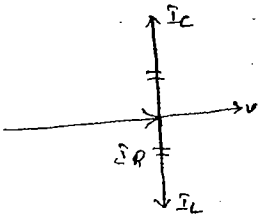
$$I = \sqrt{I_R^2 + (I_L - I_C)^2}$$

$$\phi = \tan^{-1}\left(\frac{I_L - I_C}{I_R}\right)$$

$$\cos\phi = I_R/I \text{ (Leading)}$$

Case iv:

$X_L = X_C \Rightarrow I_L = I_C$



$$I = I_R$$

$$\phi = 0$$

$$\cos\phi = 1 \text{ upF}$$

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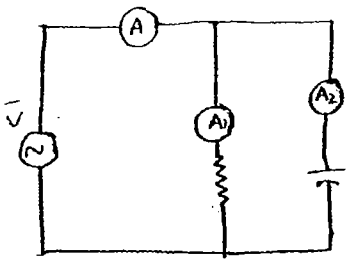
(P)



A1 reads 8A ✓

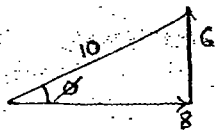
A2 reads 6A ✓

Then ckt power factor _____

$I = \sqrt{I_R^2 + I_C^2} = \sqrt{64+36} = 10A$

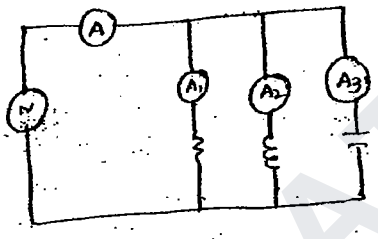
$\phi = \tan^{-1}\left(\frac{I_C}{I_R}\right) = \tan^{-1}\left(\frac{6}{8}\right) = \tan^{-1}(3/4)$ ✓

$\frac{6}{8}\ \frac{3}{4}$



$\cos\phi = \frac{8}{10} = 0.8$ lead

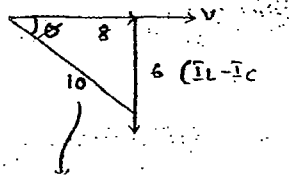
(P)



$A_1 = 8A$

$A_2 = 12A$

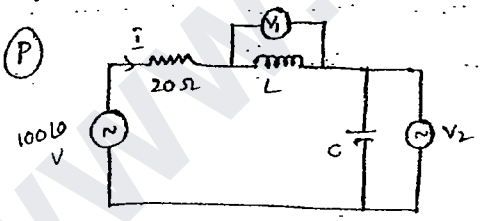
$A_3 = 6A$



$A = 10A$

$\cos\phi = \frac{8}{10} = 0.8$

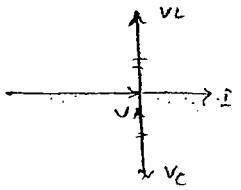
(P)



if $V_1 = V_2$

then $I = 5\angle 0$

Ckt p.f = 1



$I = \frac{V}{R} = \frac{100\angle 0}{20}$

$= 5\angle 0 A$

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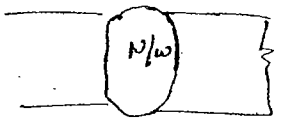
RMS value / effective value / True :

Heat cause effect

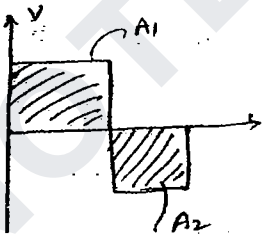


$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T (v(t))^2 dt}$$

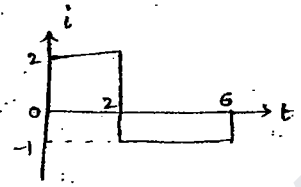
Concept of asymetrical & Symmetrical waveforms:

if $|A_1| = |A_2|$ - Symmetrical waveforms

$|A_1| \neq |A_2|$ — Asymmetrical waveforms



Ex:



$|A_1| = |A_2|$

Symmetrical

Average / Mean value:

2S — 1coulomb

Time charge

Symmetrical waveform

$$V_{avg}: \begin{cases} 0 \longrightarrow \text{full cycle} \\ \frac{1}{T/2}\int_0^{T/2} v(t)\,dt \longrightarrow \text{Half cycle} \end{cases}$$

(b) Asymmetrical waveform

$$V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt$$

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Peak factor:- / Crest factor):

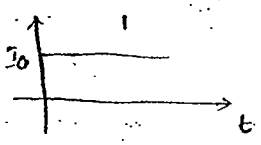
$$\text{peak factor} = \frac{\text{Maxm value}}{\text{Rms value}}$$

Form Factor:-

$$\text{Form factor} = \frac{\text{Rms value}}{\text{Avg value}}$$

peak - peak value:

difference b/w Maxm & minm value.

① 

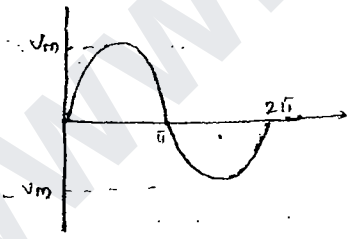
$I_{Rms} = I_0$

$I_{avg} = I_0$

peak factor = 1

Form factor = 1

② $V = V_m \sin\omega t$



$V_{Rms} = V_m/\sqrt{2}$

$V_{avg} \rightarrow 0$ (full cycle)

$\rightarrow \frac{2V_m}{\pi}$ (Half cycle)

$$\text{peak factor} = \frac{V_m}{V_m/\sqrt{2}} = \sqrt{2}$$

$$\text{form factor} = \frac{\pi}{2\sqrt{2}} = 1.11$$

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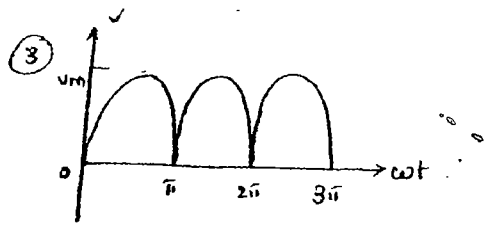
③



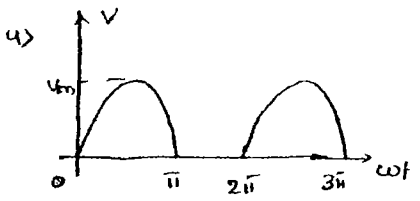
$V_{RMS} = V_m/\sqrt{2}$

$V_{avg} = 2V_m/\pi$

Peak factor $= \sqrt{2}$

Form factor $= \frac{\pi}{2\sqrt{2}} = 1.11$

④



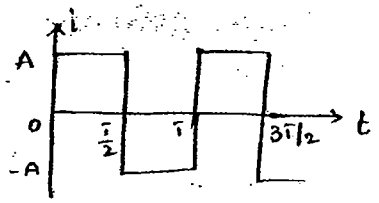
$V_{RMS} = V_m/2$

$V_{avg} = V_m/\pi$

Peak factor $= V_m/(V_m/2) = 2$

form factor $= \pi/2 = 1.57$

5.



$I_{RMS} = A$

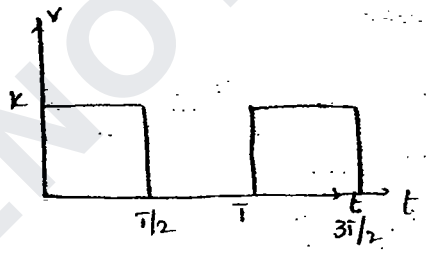
I_{avg} → 0 — Fullcycle

I_{avg} → A — Halfcycle

Peak factor = 1

Form Factor = 1

6.



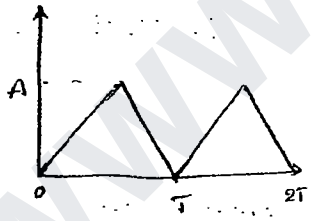
$V_{rms} = K/\sqrt{2}$

$V_{avg} = K/2$

peak factor $= \frac{K}{K/\sqrt{2}} = \sqrt{2}$

Form factor $= \sqrt{2}$

7.



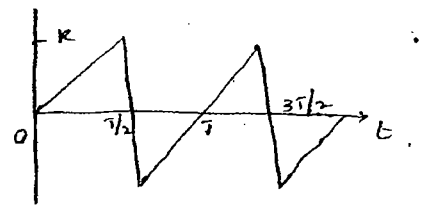
$I_{RMS} = A/\sqrt{3}$

$I_{avg} = A/2$

peak factor $= \sqrt{3}$

Form factor $= 2/\sqrt{3}$

⑧



$V_{RMS} = K/\sqrt{3}$

V_{avg} → 0 → Fullcycle

V_{avg} → $\frac{K}{2}$ — Halfcycle

Peak Factor $= \sqrt{3}$

Form Factor $= 2/\sqrt{3}$

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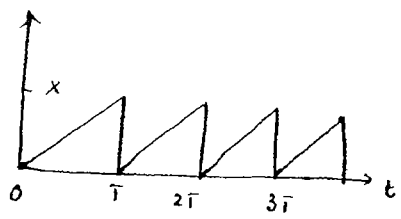
9.



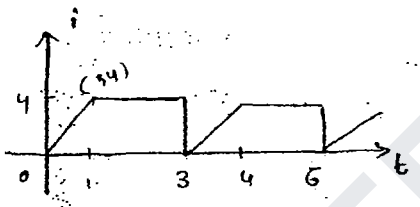
$I_{Rms} = X/\sqrt{3}$

$I_{avg} = X/2$

Peak factor: $\sqrt{3}$

Form factor: $2/\sqrt{3}$

(P) if Current through a 5Ω Resistor is as shown below



$I_{Rms} = \sqrt{\frac{1}{T}\int_0^T i(t)^2\,dt}$

→ Time period = 3sec

$(y-0) = \frac{4}{1}(x-0)$

$y = 4t$

$i(t) = \begin{cases} 0<t<1, & i(t) = 4t \\ 1<t<3, & i(t) = 4 \end{cases}$

$I_{Rms} = \sqrt{\frac{1}{3}\int_0^3 (i(t))^2 dt}$

$= \sqrt{\frac{1}{3}\left[\int_0^1 (4t)^2 dt + \int_1^3 4^2 dt\right]}$

$= \sqrt{\frac{1}{3}\left(\frac{16}{3} + 32\right)}$

$I_{Rms} = 3.52$ Amps

power lost: $I_{Rms}^2 \cdot R = (3.52)^2 \times 5$

NOTE:-

$v(t) = V_0 + V_1 \sin\omega t + V_2 \sin 2\omega t + V_3 \sin 3\omega t$

$V_{avg} = 0$

$V_{rms} = \sqrt{(V_0)^2 + \left(\frac{V_1}{\sqrt{2}}\right)^2 + \left(\frac{V_2}{\sqrt{2}}\right)^2 + \left(\frac{V_3}{\sqrt{2}}\right)^2}$

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(P) If a current of $i = 5 + 2\cos(\omega t + 30^\circ) + 4\cos(3\omega t - 30^\circ)$ is given to 10Ω resistor find power lost in it

$$i_{rms} = \sqrt{5^2 + \left(\frac{2}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2} = \sqrt{25 + 2 + 8} = \sqrt{35}$$

$$i^2R = (\sqrt{35})^2 \times 10 = 350 \text{ watts}$$

$\Rightarrow$ So

$Z = R \pm jX$

$\rightarrow Z = R + jX_L$

$\rightarrow Z = R - jX_C$

Ex: $Z = 2 + j3\ \Omega$

$\frac{1}{Z} = Y \rightarrow$ Admittance ($\mho$)

or Siemens

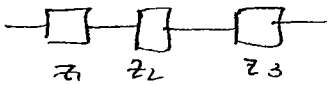
So $Y = \frac{1}{2+j3} \times \frac{2-j3}{2-j3} = \frac{2}{13} - j\frac{3}{13}$

$\Rightarrow$

$Y = G \pm jB$

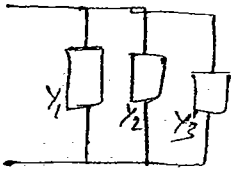
$\rightarrow Y = G - jB_L$ (where $B_L = \frac{1}{X_L} = \frac{1}{\omega L} = \frac{1}{2\pi fL}$) inductive Susceptance $\mho$

$\rightarrow Y = G + jB_C$ (where $B_C = \frac{1}{X_C} = \omega C = 2\pi fC$). Capacitive Susceptance $\mho$



Impedance in Series can be added

$Z_1 + Z_2 + Z_3$



admittance parallel can be added

$Y_2\ Y_1 + Y_2 + Y_3$

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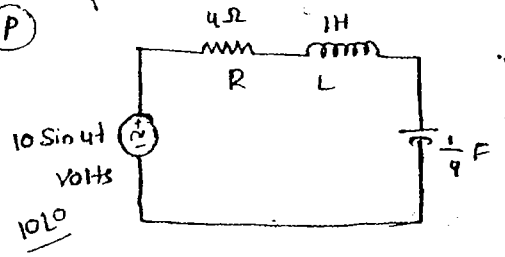
(P) Main



a> Z. e> Verify KVL

b> Y f> P, Q, S

c> I g> Conclusion

d> V_R, V_L, V_C h> phasor diagram

a

a> $\underline{Z}$

$Z = R + j(X_L - X_C)$

$= 4 + j(4-1)$

$= \underline{4+j3} = \underline{4+3j}$

$X_L = 2\pi f L$

$= j2\pi \times \frac{4}{2\pi} \times 1H$

$= \underline{4j}$

10 Sin ωt

$\omega = 4$

$2\pi f = 4$

$f = 4/2\pi$

$X_C = \frac{1}{j2\pi fc} = \frac{1}{2\pi \times \frac{4}{2\pi} \times \frac{1}{4}} = \frac{1}{j} = \underline{-1j}$

Reactance form:

(i) $Z = 4+3j$

$|Z| = \sqrt{4^2+3^2} = \underline{5\Omega}$ impedance.

(ii) $Y = \frac{1}{Z} = \frac{1}{4+j3} = \frac{4-j3}{16+9} = 0.16 - j0.12$ ℧

$|Y| = \frac{1}{|Z|} = \frac{1}{\sqrt{16+9}} = \frac{1}{5} = 0.2$ ℧

iii) $I = \frac{V_S}{Z} = \frac{10\angle 0}{4+j3} = \frac{10\angle 0}{5\angle 36.86} = 2\angle -36.86$

$I_{rms} = \frac{I}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2} = 1.414\angle -36.86$ A.

(4)

$V_R = I_R \cdot R$

$= 1.414\angle -36.86 \times 4 = 5.656\angle -36.86$ V

Current lags voltage

5. V_L $V_L = I_L[+jX_L] = [1.414\angle -36.86][j4]$

$= [1.414\angle -36.36][4\angle 90]$

$= 5.656\angle 53.14$ V

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$$V_C = I_C[-jX_C]$$

$$= [1.414 \angle -36.86][-j1]$$

$$= [1.414 \angle -36.86][1 \angle -90]$$

$$= 1.414 \angle -126.86^\circ \ V$$

(e) Verify KVL

$$-V_S + V_R + V_L + V_C = 0 \quad (\text{Rms only})$$

$$V_S = V_R + V_L + V_C$$

$$V_{S(RMS)} = \frac{10}{\sqrt{2}} \angle 0^\circ = 7.07 \angle 0^\circ \ [\text{L.H.S}].$$

$$\text{R.H.S} = 5.656 \angle -36.86 + 5.656 \angle 53.14 + 1.414 \angle -126.86$$

$$= 7.07 \angle 0^\circ = 7.07 + j0 \ V$$

So KVL is Verified

(f) Find P, Q, S

while Calculating power we Consider RMS magnitudes only

$$\Rightarrow P = I_R^2 R = \frac{V_R^2}{R} = V_R \cdot I_R$$

$$= (1.414)^2 4 = \frac{(5.656)^2}{4} = (5.656)(1.414) = 8 \text{ watts.}$$

$$\Rightarrow Q = Q_{net} = I^2 X_{net} = \frac{V^2}{X} =$$

$$= (1.414)^2 (3) = \underline{6 \ V}$$

Q_{net} — Q_L, $-Q_C$

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$$+Q_L = I_L^2 X_L = \frac{V_L^2}{X_L} = (1.414)^2 4 = \frac{(5.656)^2}{4} = (5.656)(1.414)$$

$$= V_L \cdot I_L$$

$\therefore$ 8 VARS

$$-Q_C = I_C^2 X_C = \frac{V_C^2}{X_C} = V_C \cdot I_C$$

$$= -\left[(1.414)^2(1) = \frac{(1.414)^2}{1} = (1.414)(1.414)\right.$$

$Q_C = -2$ Vars

$Q_{net} = Q_L - Q_C = 8 - 2 = \underline{6v}$

S

$$S = V_S I_S = I_S^2 Z = V_S^2/Z$$

$$= (7.07)(1.414) = (1.414)^2(5) = \frac{(7.07)^2}{5} = \underline{10} \text{ VAR'S}$$

P.f

power factor

$$\cos\phi = \cos(V_S, I_S) = \frac{R}{Z} = \frac{P}{S}$$

$$\cos(36.86) = \frac{4}{5} = \frac{8}{10} = 0.8 \text{ lagging}$$

$$|S| = \sqrt{P^2+Q^2}$$

$$P = S\cos\phi = 10(0.8) = 8v$$

$$Q = S\sin\phi = 10(\sin(36.86)) = 6v$$

Checking

$$|S| = \sqrt{8^2+6^2} = \underline{10}$$

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$V_S = 7.07 \angle 0^\circ$ V

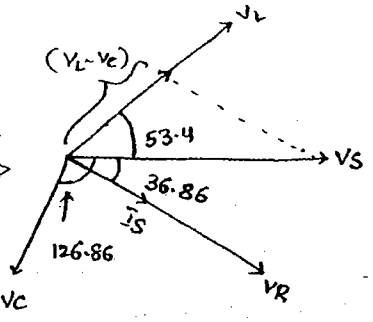
$I_S = 1.414 \angle -36.86$ A

$V_R = 5.656 \angle -36.86$

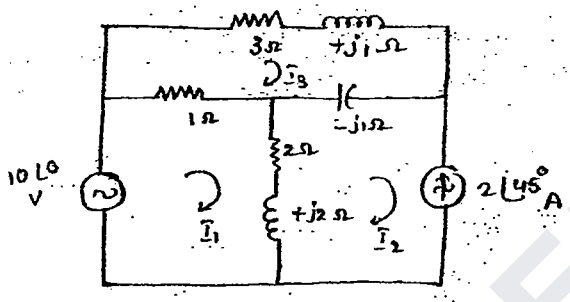
$V_L = 5.656 \angle 53.4$

$V_C = 1.414 \angle 126.86$

Look ⟹



Mesh and Nodal Analysis Ac :-



Nodes - 6

Branches - 8

meshes 8-6+1

= 3 loops

$$-[10\angle 0] + [I_1 - I_3] + [2+j2][I_1 - I_2] = 0$$

$$I_1[3+j2] - [2+j2]I_2 - I_3 = 10 \quad \text{—— ①}$$

$$I_2 = -[2\angle 45^\circ] \quad \text{—— ②}$$

$$(3+j1)I_3 - j1[I_3 - I_2] + [I_3 - I_1] = 0$$

$$-I_1 + [j1]I_2 + 4I_3 = 0 \quad \text{— ③}$$

$$\begin{bmatrix} 3+j2 & -[2+j2] & -1 \\ 0 & 1 & 0 \\ -1 & j1 & 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -[2\angle 45^\circ] \\ 0 \end{bmatrix}$$

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Nodal:

$$V_1 = 10\angle 0 \quad -①$$

$$\frac{V_2 - V_1}{1\Omega} + \frac{V_2}{2+j2} + \frac{V_2 - V_3}{-j1} = 0$$

$$(V_2 - V_1) + V_2\left(\frac{2-j2}{8}\right) + j[V_2 - V_3] = 0$$

$$8V_2 - 8V_1 + V_2(2-j2) + j8[V_2 - V_3] =$$

$$-8V_1 + (10+j6)V_2 - j8V_3 = 0 \quad \text{——} ②$$

$$-[2\angle 45] + \frac{V_3 - V_2}{-j1} + \frac{V_3 - V_1}{3+j1} = 0$$

$$j[V_3 - V_2] + \frac{(V_3 - V_1)(3-j1)}{10} = 2\angle 45^\circ$$

$$V_1[-3+j1] - j10V_2 + (3+j9)V_3 = 20\angle 95^\circ \quad \text{——} ③$$

Matrix:

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ -8 & 10+j6 & -j8 \\ -3+j1 & -10j & 3+j9 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10\angle 0 \\ 0 \\ 20\angle 45^\circ \end{bmatrix}$$

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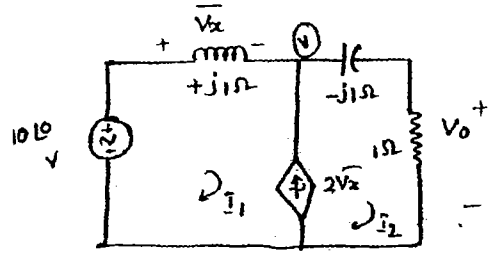
A|C Network theorems:-



- find Vo using mesh & Nodal.

Mesh Analysis:

$$-[10\angle 0] + (j1)\bar{I}_1 - (j1)\bar{I}_2 + 1\,\bar{I}_2 = 0 \qquad j_1\bar{I}_1 + (1-j_1)\bar{I}_2 = 10\angle 0 \quad -(1)$$

$$[j_1]\,\bar{I}_1 - [j_1]\bar{I}_2 \qquad -\bar{I}_1 + \bar{I}_2 = 2V_x \quad -(2)$$

$$V_x = \bar{I}_1[j_1] \quad -(3)$$

(3) in (2)

$$-\bar{I}_1 + \bar{I}_2 = 2\,\bar{I}_1[j_1]$$

$$-[1+j_2]\bar{I}_1 + \bar{I}_2 = 0 \quad -(4)$$

$$\bar{I}_2 = (1+j_2)\bar{I}_1$$

from (4) & (1)

$$j_1[\bar{I}_1] + (1-j_1)(1+j_2)\bar{I}_1 = 10$$

$$\bar{I}_1[j_1] + [3+j_1]\bar{I}_1 = 10$$

$$\bar{I}_1 = \frac{10}{3+j_2}$$

So $\bar{I}_2 = \left(\frac{10}{3+j_2}\right)(1+j_2)$

$$= \frac{10(1+j_2)}{3+j_2} = 6.2\angle 29.78^\circ\ V$$

$$\sqrt{r_1\angle\theta_1 \cdot r_2\angle\theta_2} = \left[\sqrt{r_1 r_2}\right]\left[\frac{\theta_1+\theta_2}{2}\right]$$

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Nodal :-

$$\left(\frac{V-10}{j1}\right) - 2V_x + \frac{V}{(1-j1)} = 0$$

$$-j[V-10] - 2V + V\left[\frac{1+j1}{2}\right] = 0$$

$$-j2V + j20 - 4V_x + V[1+j1] = 0$$

$$V[1-j] - 4V_x = -j20 \quad \text{—— ①}$$

$$V_x = 10 - V \quad \text{—— ②}$$

$$V[1-j] - 4(10-V) = -j20$$

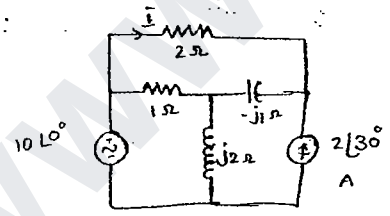
$$V = \frac{40-j20}{5-j} \times \frac{1}{1-j}$$

$$V_0 = \frac{40-j20}{5-j} \times \frac{1}{1-j}$$

So, $\bar{I}_2 = \left[\frac{10}{3+j2}\right](1+j2)$

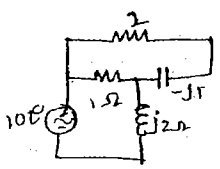
$$V_0 = 1\cdot\bar{I}_2 = \frac{10[1+j2]}{(3+j2)} = 6.2\angle 29.78^\circ \text{ V}$$

Ex:



find $\bar{I}$ using ① S.P.T ② Thevenin's ③ Norton's & Verify the relation with mesh & Nodal.

S.P.T when 10V only



$$Z_{eq} = 0.7 + j1.1$$

$$\bar{I}' = \frac{10\angle 0^\circ}{0.7+j1.9}\left[\frac{1}{3-j1}\right] = 1.56\angle -51.3$$

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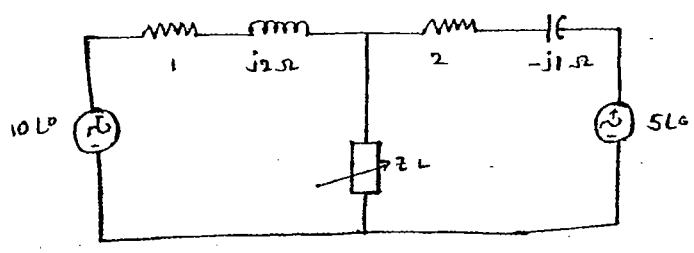
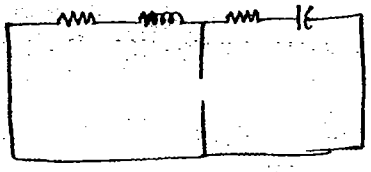
(Pb)



find R_{th}

& P_{max}

Sol.



$$\frac{(1+j_2)(2-j_1)}{1+j_2+2-j_1} = 2.$$

$$= 1.57\angle 18.41 = 1.5+j0.5$$

$$P_{max} = \frac{V_{th}^2}{4R_{th}}$$

By Nodal analysis $\frac{V-10}{1+j} + \frac{V-5}{2-j_1} = 0$ $\quad V = 7.91\angle -18.4^\circ$

$$P_{max} = \frac{(7.91)^2}{4\times 1.5} = 10.42 \text{ watts}$$

* Concept of Complex power

Mathematical way to remove the confusion in Calculating power in any Element ie Neglecting the lead & lag concept

Case(i)

$V = V\angle\alpha$

$I = I\angle\beta$

$\alpha > \beta$

$VI = V\angle\alpha \; I\angle\beta$

$= VI\angle\alpha+\beta$ X

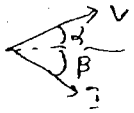
$VI^* = V\angle\alpha \cdot I\angle-\beta$

$= VI\angle\alpha-\beta$

Case(ii)

$V = V\angle\alpha$

$I = I\angle-\beta$



$VI = VI\angle\alpha-\beta$ X

$VI^* = VI\angle\alpha+\beta$

So complex power

$S^* = VI^*$

Unit VA

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dual and duality:-

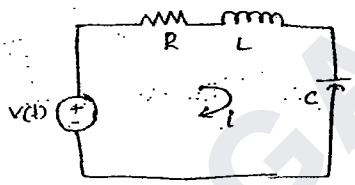
dual Two ckts are said to be duals one each other if the Mesh eq that characterised one of them as the given mathematical form as the Nodal Eq'n characterized

Exact dual: Two ckts are exacts duals if mesh equation of one and the Nodal Equation of the other are Numerically Identical.

Duality:- Identical behaviour patterns observed b/w voltages and Curren of two ckts to illustrate the principle of duality.

Note: In duality Concept we use Exact Equations

Ex: Series R-L-C circuit

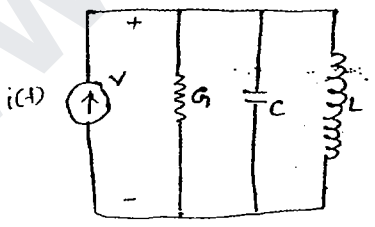


$\Rightarrow$ Mesh Equation $\longrightarrow$ KVL

$$-V(t) + iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0$$

$$V(t) = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0 \quad \text{— (A)}$$

Parallel G-C-L ckt



$\Rightarrow$ Nodal Equation $\longrightarrow$ KCL

$$i(t) = VG + C\frac{dv}{dt} + \frac{1}{L}\int v\,dt \quad \text{— (B)}$$

The Equations A & B are mathematically of the same form Hence these two circuits are duals of each other

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Dual. duality is very important in control system as we study Analogous Systems

Basic dual elements:

$i(t) \longleftrightarrow v(t)$

$I \longleftrightarrow V$

$V_m \sin\omega t \longrightarrow I_m \sin\omega t$

$R \longleftrightarrow G$

$L \longleftrightarrow C$

KVL $\longleftrightarrow$ KCL

Mesh $\longleftrightarrow$ Nodal

Mesh analysis $\longleftrightarrow$ Nodal analysis

Series $\longleftrightarrow$ parallel

Switch open $\longleftrightarrow$ Switch closed

$Z \longleftrightarrow Y$

$X \longleftrightarrow B$ ✓

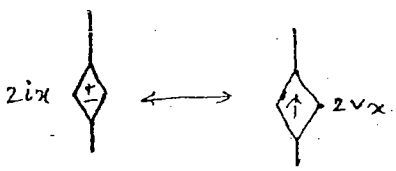
$Y \longleftrightarrow \Delta$

$\overline{\Pi} \longleftrightarrow \overline{T}$

$\phi \longleftrightarrow q$

Link (chords) $\longleftrightarrow$ twig

o.c $\longleftrightarrow$ S.c



Theorems $\longleftrightarrow$ Nortons

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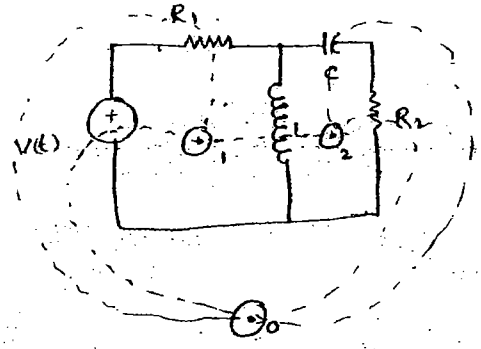
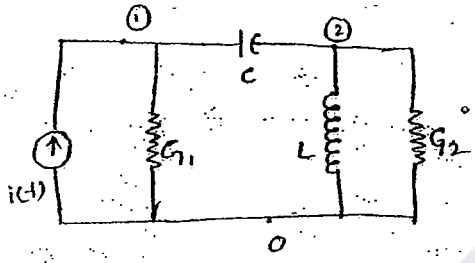
Steps to write duality of a given N/w:

(P) write the dual of the given ckt & verify with Equations



$$\Rightarrow -v(t) + i_1 R_1 + L\left[\frac{di_1}{dt} - \frac{di_2}{dt}\right] = 0$$

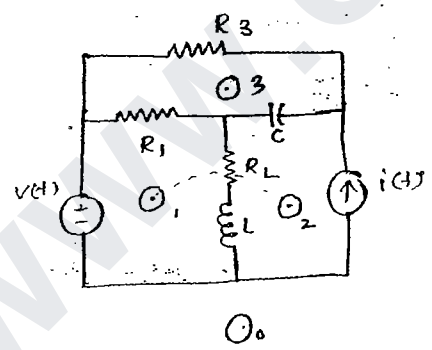
$$L\left[\frac{di_2}{dt} - \frac{di_1}{dt}\right] + \frac{1}{C}\int i_2\,dt + i_2 R_2 = 0$$

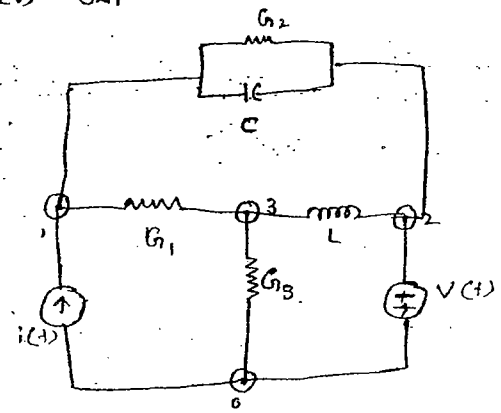


$$\Rightarrow -i(t) + v_1 G_1 + C\left[\frac{dv_1}{dt} - \frac{dv_2}{dt}\right] = 0 \quad \text{— 1(B)}$$

$$C\left[\frac{dv_2}{dt} - \frac{dv_1}{dt}\right] + \frac{1}{L}\int v_2\,dt + v_2 G_2 = 0 \quad \text{— 2(B)}$$

(P) write the dual of the given ckt





$$-v(t) + i_1 R_1 + R_2(i_1 - i_2) + L\left[\frac{di_1}{dt} - \frac{di_2}{dt}\right] = 0$$

$$i_2 = -i(t)$$

$$R_3 i_3 + R_1(i_3 - i_2) + \frac{1}{C}\int (i_3 - i_1)\,dt = 0$$

Nodal

$$-i(t) + G_1[v_1 - v_3] + G_2[v_1 - v_2] + C\left[\frac{dv_1}{dt} - \frac{dv_2}{dt}\right] = 0 \quad \text{— 1(B)}$$

$$v_2 = -v(t) \quad \text{— 2(B)}$$

$$G_1[v_3 - v_1] + G_3 v_3 + \frac{1}{L}\int v_3 - v_2 = 0 \quad \text{— 3(B)}$$

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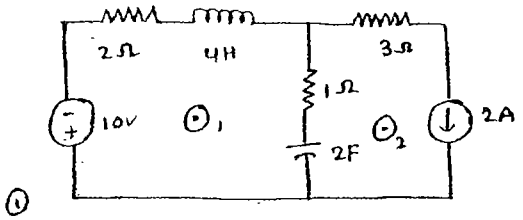
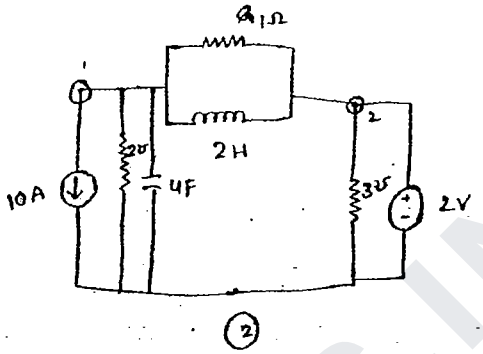
(P)





$$10V + 2i_1 + 4\frac{di_1}{dt} + \frac{1}{2}\int I_1 - I_2 + (I_1 - I_2) = 0 \quad \text{—— (1)A}$$

$$i_2 = 2A \longrightarrow (2)(B)$$

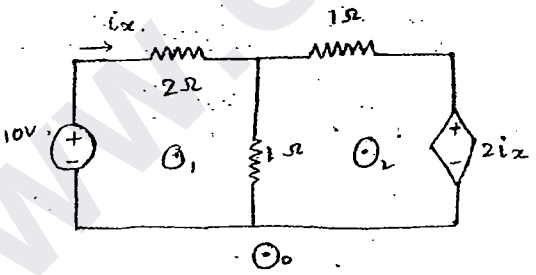
from ckt (2)

Apply Nodal Analysis

$$+10 + 2v_1 + 4\frac{dv_1}{dt} + [v_1 - v_2] + \frac{1}{2}\int v_1 - v_2\, dt = 0 \quad \text{(A)}$$

$$V_2 = 2 \quad \text{(B)}$$

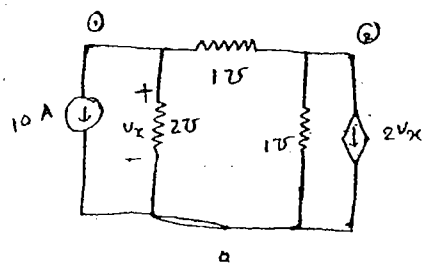
(P) determine dual of Network



$$-10 + 2I_1 + 1[I_1 - I_2] = 0 \quad -(1)$$

$$[I_2 - I_1] + I_2 + 2i_x = 0 \quad -(2)$$

$$i_x = I_1 \quad \text{—— (3)}$$



$$-10 + 2v_1 + (v_1 - v_2) = 0 \quad —(1)$$

$$(v_2 - v_1) + v_2 + 2v_x = 0 \quad —(2)$$

$$v_x = v_1 \quad —(3)$$

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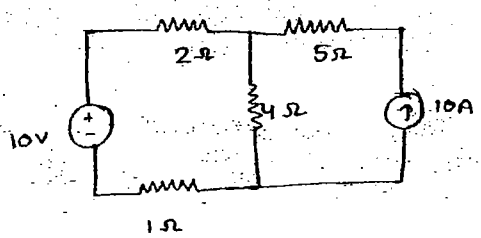
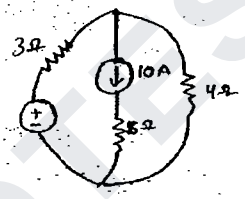
Network topology and Graph theory:

Topology: It is the Branch of Geometry applicable to electrical ckt analysis where even by bending, twisting, swaping, making ckt up & down, stretching, tieing knots will not disturb the ckt properties

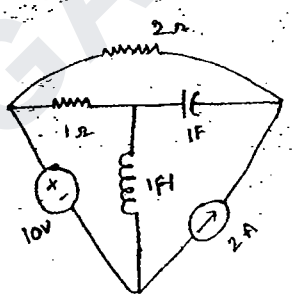
Ex:

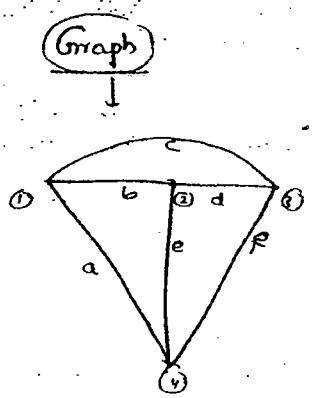




Graph: Graph is a skeleton representation of N/w or ckt where every element is suppressed in its properties & represented as a simple line segment

Ex:





Note:

Nodes are numbered ①, ②, ③ ...

Branches are named a, b, c ...

Node n: Node is a point of interconnection (or) junction b/w two or more components.

Branch B Elemental connection b/w two nodes ie every element by itself is a branch

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Path: It is a traversal from one node to another node without crossing the same node twice.

Mesh: Mesh is a closed path which should not have further closed paths in it.

$$m = b - n + 1$$

Meshes are called independent loops.

Loop: It is a closed path of ckt or N/w which may have further closed paths in it.

Ex:

$n = 4$

$b = 6$

$m = b - n + 1 = 6 - 4 + 1 = 3$

$m_1 \rightarrow ① ② ④ ①$

$m_2 \rightarrow ② ③ ④ ②$

$m_3 \rightarrow ① ③ ② ①$

Ex: No. of paths from ① to ③

① ⟶ ③

① → ② → ③

① → ④ → ③

① → ④ → ② → ③

① → ② → ④ → ③

Ex: Loops

L_1 - ① ③ ④ ①

L_2 - ① ② ③ ① ①

L_3 - ① ③ ② ④ ①

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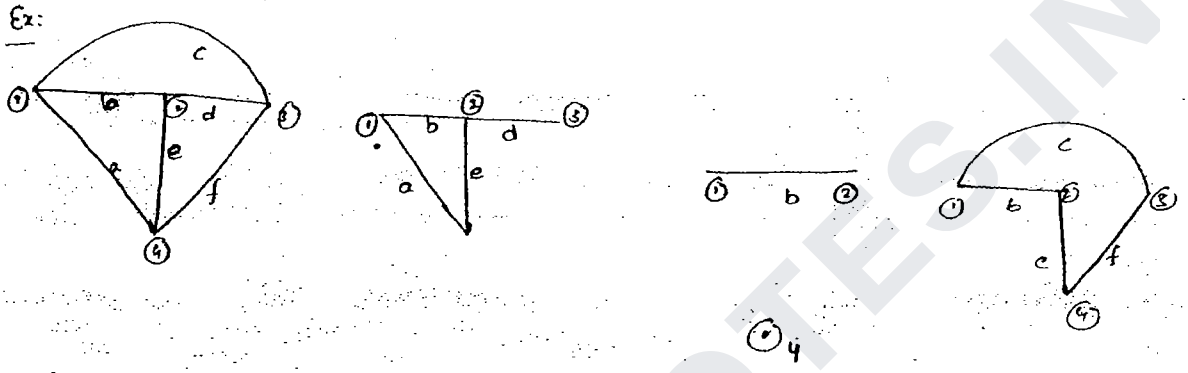
Sub graph:-

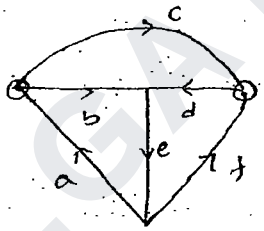
A Sub graph Consist of Some of elements & Nodes of the Main graph.

Ex:



oriented graph: if every branch in a graph is given a particular orientation or direction. which need not necessarily be the Current direction then it is said to be oriented graph.

Ex:



Connected graph:

a graph is Said to be Connected if there is Exist one path from Every node to every other node

Completely Connected graph:

if there is a direct path from Every node to Every Other node

NOTE: ✓ The No of edges in a Completely Connected graph with n nodes is $\frac{n(n-1)}{2}$

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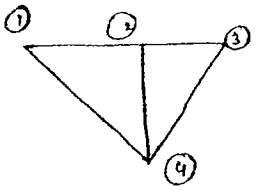
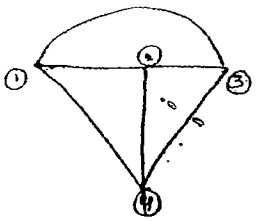
Ex:



just Connected graph

Since there is no direct path from ① to ③



Completely Connected graph.

1, ——— : $\frac{2(2-1)}{2} = \frac{1}{=}$

2, △ : $\frac{3(3-1)}{2} = 3$

3 : $\frac{4(4-1)}{2} = \frac{6}{=}$

4 : $\frac{5(5-1)}{2} = 10$

Tree: Tree is a Sub graph of main graph which Connects all the nodes with out forming closed loops.

NOTE: Rank of a tree with n nodes is $n-1$

No of trees for a given graph

- for Completly Connected graph $= n^{n-2}$ for $n>2$

  where n is no of nodes

For just Connected graph = $\det |[A_r][A_r]^T|$

A_r – Reduce incidance matrix

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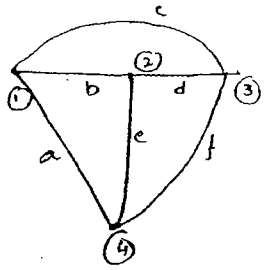
(P) The no of trees for the graph below is Equal — & identify them



— Completely Connected $= n^{n-2}$

$= (4)^{4-2} = 16$

Twig: The branch of a tree is called twig.

A tree with n nodes has n-1 twigs.

Co-tree or Complement of a tree: The set of branches other than tree form a Cotree

Link or Chord: The Branches of Cotree are Called as Links. for any graph with n nodes & branches the no of Links.

no of Links $= b - n + 1$

$= b - (n - 1)$

NOTE: Twigs are indicated by thick lines

Links are indicated by dotted lines

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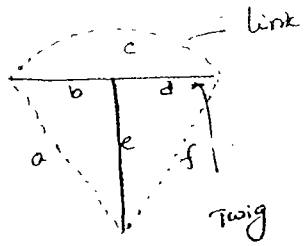
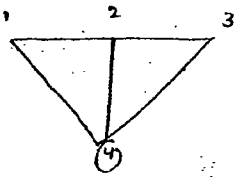
Ex:



bde → Tree

acf → Cotree

degree of a node: The no of branches incident at any node Represents its degree



degree of nod ② - 3

degree of node ④ - ③

Incidance matrix: [A]:

Matrix which gives the Relation Ship btw No of nodes and no of branches & the orientation of a Branch w.r.to the incident node

The order of incidance matrix is → n x b

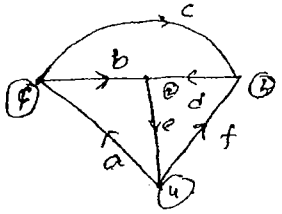
$[A] = [a_{ij}]_{n \times b}$ where $a_{ij} = +1$, if jth branch is incident on ith node & oriented away from it.

$a_{ij} = -1$ jth branch is incident on ith node & oriented towards from it

$a_{ij} = 0$ jth branch not incident with ith node

④ Construct the N/w graphs incidance matrix for N/w graph

Ex:



	a	b	c	d	e	f
①	-1	+1	+1	0	0	0
②	0	-1	0	-1	1	0
③	0	0	-1	1	0	-1
④	1	0	0	0	-1	+1

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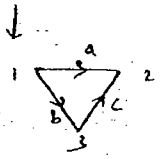
observation: The algebric Sum of elements of every column in a complete incidance matrix is zero.

✓ Rank of n nodes in incidance matrix n-1

✓ The determinent of incidance matrix of a closed loop graph — 0



$$|A| = \begin{vmatrix} & a & b & c \\ ① & 1 & 1 & 0 \\ ② & -1 & 0 & -1 \\ ③ & 0 & -1 & 1 \end{vmatrix} = 1[-1] - 1[-1]$$

$$= -1+1 = 0$$

Reduced incidance Matrix Ar:

if one of the node is ca in the graph Consider as reference node & that particular ~~no~~ row is neglected then it is a Reduced incidance Matrix.

For the above Example if node (4) is Reference node 4th row Can be neglected so the reduced incidance matrix Ar:

$$A_r = \begin{matrix} a & b & c & d & e & f \end{matrix}$$

$$\begin{matrix} -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & +1 & 0 \\ 0 & 0 & -1 & +1 & 0 & -1 \end{matrix}$$

NOTE:

→ In a Reduced incidance matrix the algebric Sum of Sum of the columns is $\neq 0$

→ In Computer methods of Electrical Systems reduced incidance matrix will Reduce the Size of the matrix. There by memory requirements & Computation time is less.

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Isomorphism: if the incidance matrix of two graphs are Exactly Same then they are Said to obey principle of Isomorphism

(P) Develop the oriented graph if incidance matrix as given below.

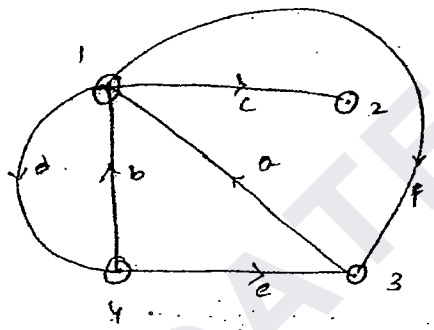
(P)
$$\begin{bmatrix} -1 & -1 & 1 & 1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ +1 & 0 & 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \end{bmatrix}$$

un Completed

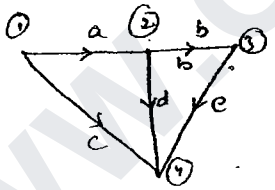
Completed

Given is reduced incidance matrix it Complete So node - 4

Branches - 6



(P) determine the no of trees for the oriented graph below.



No of trees - $\left| [A_r][A_r]^T \right|$

$$A_r = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{bmatrix} a & b & c & d & e \\ 1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & -1 & -1 \end{bmatrix} \times$$

$$|A_r| \times |A_r|^T = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[A_r][A_r]^T = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$|A_r A_r^T| = 2(6-1) + (-2-0) + 0(\ )$$
$$= 10 - 2$$
$$= 8$$

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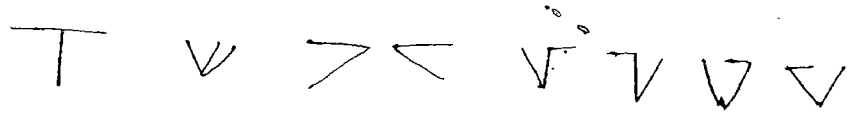
possible trees = 8



Concept of fundamental loops:

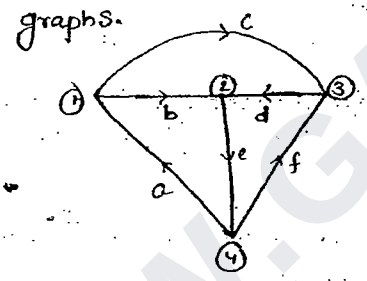
→ It is a closed path of a ckt with only one link and rest of them as twigs

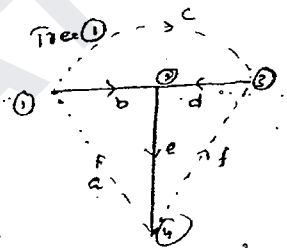
→ This fundamental loop currents are called tie set currents.

→ The orientation of these tieset currents is same as the link governing loop

→ The No of fundamental loops = no of link = b − (n−1).

Ex: write the fundamental loop currents for the following oriented graphs.



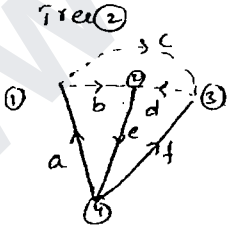


No of f-loops = 3

fl_1 − a be → i_1

fl_2 − bcd → i_2

fl_3 − def → i_3



No of f-loops = 3

fl_1 − bea − i_1

fl_2 − def − i_2

fl_3 − acf → i_3

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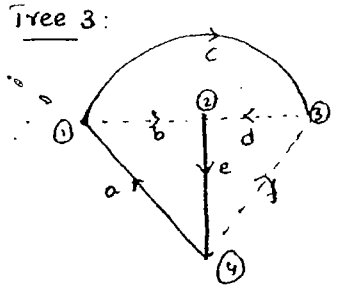
Tree 3:

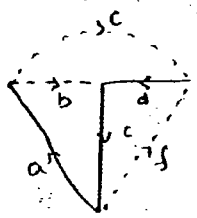


fl_1 - abe - i_1 ↑

fl_2 - acf - i_2 ↻

fl_3 - acde → i_3 ↻

Tree-4



The matrix which gives the Relation ship b/w the branch Currents & the tieset Current [M]

→ Every branch Current Can be Expres in terms of tieset Currents

→ The order of tieset matrix is (Links x branches)

$(b-n+1) \times b$

→ The elements of tieset matrix $M = a_{ij}$ $(l \times b)$.

where $a_{ij} = +1$, if the jth branch Current is incident & oriented in the Same direction of ith tieset Current.

$a_{ij} = -1$, if j^{th} branch Current is incident but oriented in opposite of ith tieset

$a_{ij} = 0$ if j^{th} branch Current not at all related to i^{th} tieset Current.

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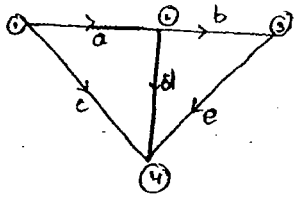
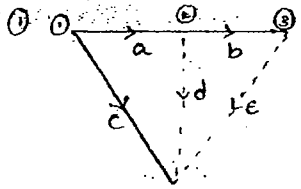
(P) Construct the tieset matrix & write the Equilibrium Eqn for the N/w graph shown below by Considering a, b, c as tree branches.



Tie set:



No of f loops = 2

$f_1 \longrightarrow a, c, d \longrightarrow i_1$

$f_2 \longrightarrow a, b, c, e \longrightarrow i_2$

(Link X Branches)

$$[M] = \begin{array}{c} \\ i_1 \\ i_2 \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 1 & 0 \\ 1 & 1 & -1 & 0 & 1 \end{bmatrix} \end{array}$$

Equilibrium Equation:

Let V_a, V_b, V_c, V_d, V_e — branch voltages.

Let j_a, j_b, j_c, j_d, j_e — branch Currents

Set - I] Horizontal [⟶] KVL

$$+V_a - V_c + V_d = 0$$

$$V_a + V_b - V_c + V_e = 0$$

Set II] Relation b/w branch Current & tieset Current.

Vertical [↓]

$j_a = i_1 + i_2$ $\quad j_d = i_1$

$j_b = i_2$ $\quad j_e = i_2$

$j_c = -i_1 - i_2$

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Concept of Cut Set, Fundamental Cut set:

Cutset: A Cut Set is a Cut through of a graph which Can devide it in to two parts

(or)

A cut set is a collection of branches which Can devide the graph in to two parts where even by placing one of these Elen Branches will destroy this properity.

Fundamental Cut Set (L):

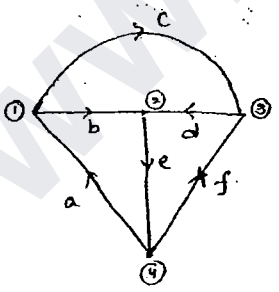
It is a Cut through of a graph which can devided in to two parts but it has to encounter only one twig and Rest of them as Links.

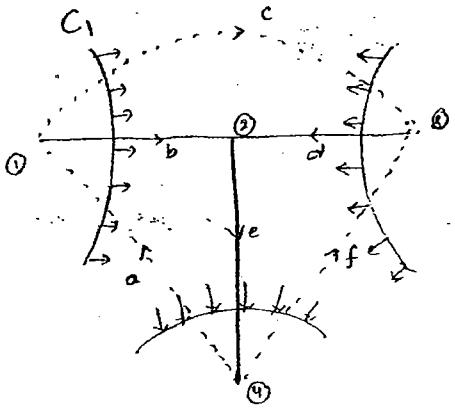
→ The No of fundamental Cut sets Equal to no of twig's ie $n(n-1)$

→ The orientation of f Cut set is Same as the orientation of twig governing it.

9/5/11

Ex:





No of f cut sets : 3

f_1 → a, b, c

f_2 → c, d, f

f_3 → a, e, f

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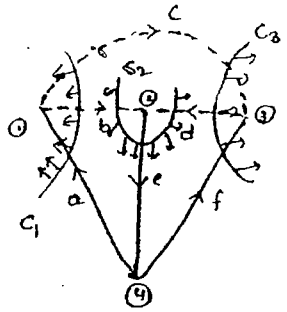
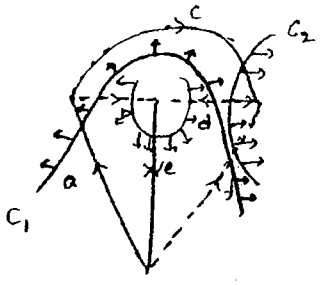
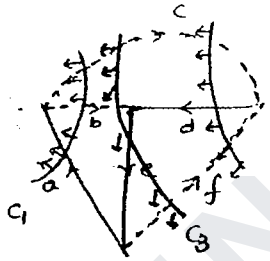






F- cut sets - 3

$fc_1 = abc \rightarrow c_1$

$fc_2 = bde \rightarrow c_2$

$fc_3 = cdf \rightarrow c_3$

F cut set 3.

$fc_1 - a,b,d,f - c_1$

$fc_2 - c,d,f - c_2$

$fc_3 - b,d,e - c_3$

F cut sets - 3

$fc_1 - abc - c_1$

$fc_2 - c,d,f - c_2$

$fc_3 - c,b,e,f - c_3$

Cut set Matrix:

Matrix which gives the Relationship b/w the Cut set voltage & branch voltage. The order of cut set is Twigs × Branches.

$(n-1) \times b$

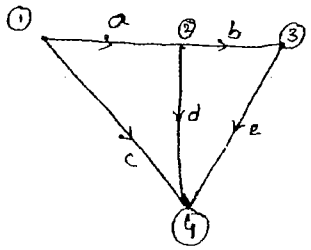
$$[c] = [a_{ij}]_{(n-1)\times b}$$

$a_{ij} = 1$ if jth branch incident to ith Cut set & oriented in Same direction

$a_{ij} = -1$ if jth branch incident to ith cut set & oriented in opposite direction

$a_{ij} = 0$ if jth branch not at all incident to ith Branch

(P) develop cut set matrix shown below by Considering a, b, c branches & write the Equilibrium Equations



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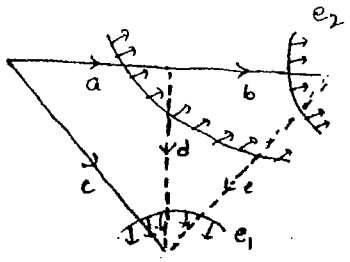
Ans



No of f cutset - 3

$fc_1 = c, d, e \quad -e_1$

$fc_2 = b\,e \quad -e_2$

$fc_3 = a\,d\,e \quad -e_3$

$$[C] = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & +1 & 0 & 0 & -1 \\ +1 & 0 & 0 & -1 & -1 \end{bmatrix} \end{array}.$$

Equilibrium equations.

Let V_a, V_b, V_c, V_d, V_e are Branch voltages

j_a, j_b, j_c, j_d, j_e are Branch Currents

Set i/ [⟶] Horizontal KCL

$$j_c + j_d + j_e = 0$$

$$j_b - j_e = 0$$

$$j_a + j_d - j_e = 0$$

Set ii/ [↓] Vertical KVL (Relation b/w Cut set voltage & Branch

$$V_a = e_3$$

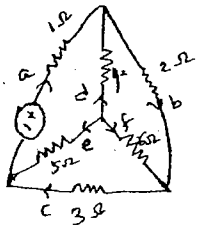
$$V_b = e_2$$

$$V_c = e_1$$

$$V_d = e_1 - e_3$$

$$V_e = e_1 - e_2 - e_3.$$

(P)



develop Cut Set & tieset matrices & write Corresponding Equilibrium Eqn. by Considering branches d, f, c as tree branches

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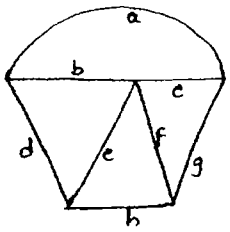
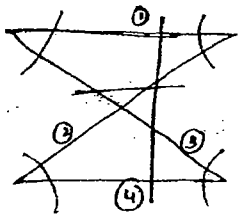
(P) which of the following is not a tree

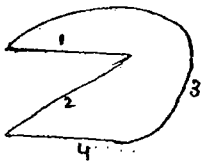


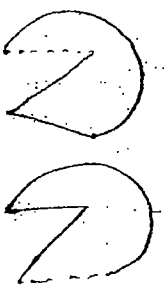
a) bcgh
b) defg
c) abfg
d) aegh

following graph No of trees P & No of Cut sets Q are



a) P = 2 Q = 2
b) P = 2 Q = 6
c) P = 4 Q = 6
d) P = 4 Q = 10





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9/08/12

Electrical Resonance.

→ Resonance is a phenomenon in electrical ckt where the applied voltage and Source Current are in phase.

→ At Resonance condition ckt behaves as purely resistive. So it is having unity p.f.

→ Resonance is the frequency Response of an electrical ckt such that at Resonant frequency the net impedance equal to resistance only. (Net Reactance = 0)

$X_{net} = 0$

$Z = R + jX$

$Z = R$

→ In any electrical ckt Resonance can occur provided we have two similar but opposite nature energy storage elements (L & C)

→ To undergo Resonance we need good inductor & good Capacitor

→ The goodness / Quality of this inductors & Capacitors is measured in terms of Quality factor Q which is also called as figure of merit

→ The Min^m Quality factor to undergo Resonance in practical ckt is $Q = 10$

→ For practical P.S applications of resonance such as tuned filter the Q-factor can be as high as 100

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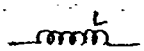
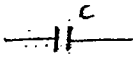
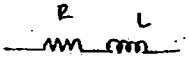
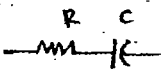
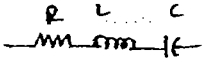
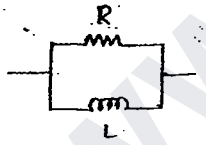
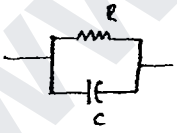
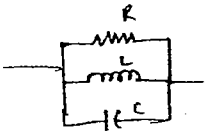
Q-Factor

$$Q\text{-Factor} = 2\pi \times \left[\frac{\text{Max}^m \text{ Energy Stored per cycle of Supply}}{\text{Max}^m \text{ Energy dissipated per cycle of Supply}} \right]$$

Element	Q factor
	$Q = 0$
	$Q = \infty$
	$Q = \infty$
	$Q = \frac{\omega L}{R}$
	$Q = \frac{1}{\omega CR}$
	$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$
	$Q = R/\omega L$
	$Q = \omega CR$
	$Q_0 = R\sqrt{\frac{C}{L}}$

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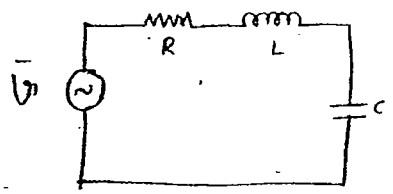
Series Resonance:-



ω ↓ frequency varied

$Z = R + jX_L - jX_C$

$= R + j(X_L - X_C)$

So at Resonance frequency $\omega = \omega_0$

net Resonance = 0

$X_{net} = 0$

$X_L - X_C = 0$

$X_L = X_C$

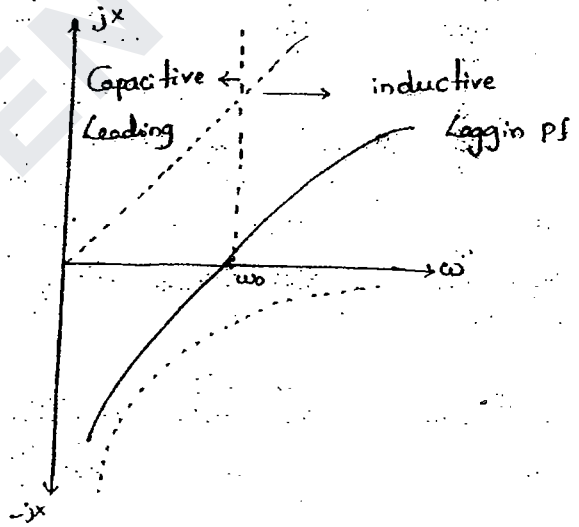
$\omega_0 L = \frac{1}{\omega_0 C}$

$\omega_0 = \sqrt{\frac{1}{LC}}$ rad/sec

$f_0 = \frac{1}{2\pi\sqrt{LC}}$ Hz

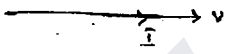
Reactance diagram:

$Z = R + j\left[\omega L - \frac{1}{\omega C}\right]$



(i) at $\omega = \omega_0$

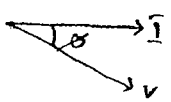
$Z = R$



$\phi = 0$

ii) $\omega < \omega_0$

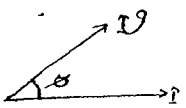
$Z = R - jX$



$\phi \rightarrow -ve$

iii) $\omega > \omega_0$

$Z = R + jX$



ϕ is +ve

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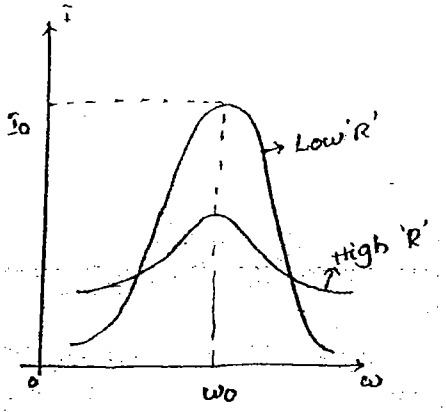
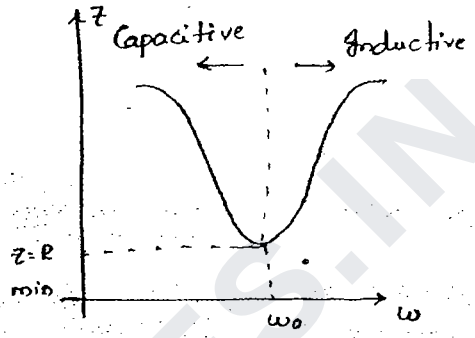
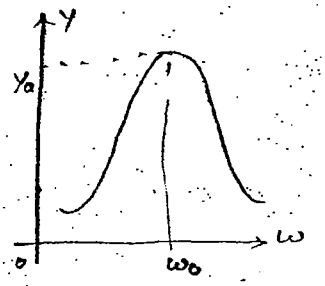
I (vs) ω :-



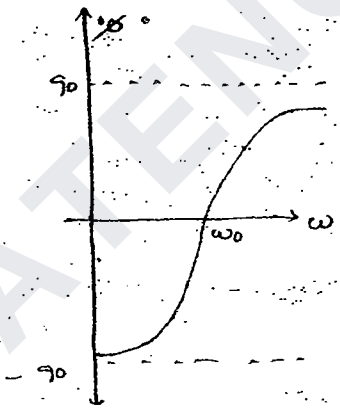
Z (vs) ω :-



Y (vs) ω :-



Ø (vs) ω :



Q Factor at Resonance frequency:

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

but $\omega_0 = \frac{1}{\sqrt{LC}}$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

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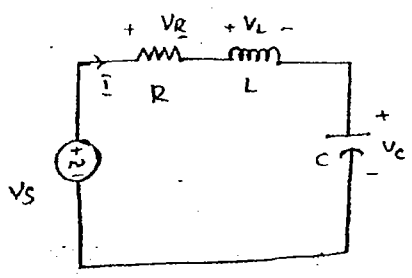
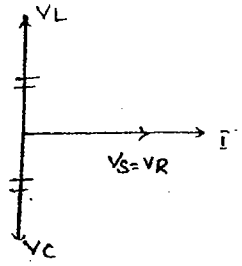
Complete phasor diagram:





- Complete Source voltage get applied across Resistance

→ during Series Resistance the net impedance is min^m hence it is called as acceptor ckt

→ Since the Entire Supply voltage appear across Resistor Series Resonant ckt is called as voltage aplification ckt

$$|\bar{I}| = \frac{V_S}{Z} = \frac{|V_S|}{\sqrt{R^2+(\omega L-\frac{1}{\omega C})^2}}$$

But at $\omega = \omega_0 \Rightarrow X_L = X_C$

$$I_{0\,(Max)} = \frac{|V_S|}{R}$$

→ $V_R = \bar{I} \cdot R = \frac{V_S}{R} \cdot R = V_S$

→ $V_L = \bar{I}[jX_L] = \frac{|V_S|}{R} j\omega_0 L = j|V_S| \frac{\omega_0 L}{R}$

$V_L = jV_S Q = Q V_S \angle 90^\circ$ volts

→ $V_C = \bar{I}[-jX_C] = \frac{|V_S|}{R} \frac{1}{j\omega_0 C} = \frac{V_S Q}{j}$

$V_C = Q V_S \angle -90^\circ$ volts

$V_S = V_S + QV_S\angle 90 + QV_S\angle 90 = V_S$ Verified

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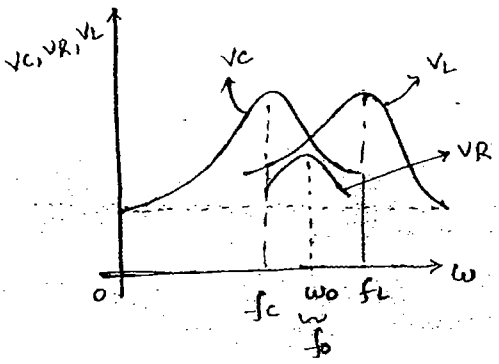
Band width:-

Variation of voltage across passive elements with ω:-



The frequency at which Maxm voltage appear across occurs

across Capacitor $f_c = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}$

$= \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{R^2C}{2L}}$

$f_c = f_o\sqrt{1 - \frac{R^2C}{2L}}$

The frequency at which Maxm Voltage appear across Inductor

$f_L = \frac{1}{2\pi}\sqrt{LC - \frac{R^2C^2}{2}}$

$= \frac{1}{2\pi\sqrt{LC}}\frac{1}{\sqrt{1 - \frac{R^2C}{2L}}}$

$f_L = \frac{f_o}{\sqrt{1 - \frac{R^2C}{2L}}}$

Band width: It is Range of frequencies for which the power level in the Signal is at least half of Maxm power

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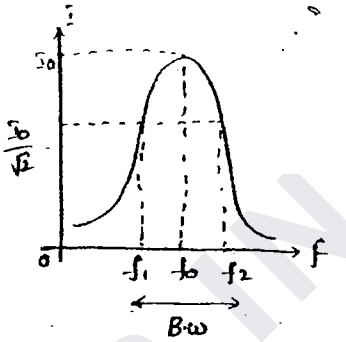
$P = I^2 R$

$P_0 = I_0^2 R \longrightarrow$ Max^m power

$\frac{P_0}{2} = \frac{I_0^2 R}{2} = \left(\frac{I_0}{\sqrt{2}}\right)^2 \cdot R$

$= (0.707\, I_0)^2 R.$



$B.\omega = f_2 - f_1 = \frac{R}{2\pi L}$

$B.\omega = \omega_2 - \omega_1 = \frac{R}{L}$

f_1 - Lower Cut off frequency

f_2 - upper Cut off frequency.

Resonance frequency is Geometric mean of Corner frequencies

$f_0 = \sqrt{f_1 f_2}$

$\omega_0 = \sqrt{\omega_1 \omega_2}$

$f_0 - f_1 = \frac{B\omega}{2} = \frac{R}{4\pi L} \Rightarrow f_1 = f_0 - \frac{B\omega}{2}$

$f_2 - f_0 = \frac{B\omega}{2} = \frac{R}{4\pi L} \Rightarrow f_2 = f_0 + \frac{B\omega}{2}$

$\omega_0 - \omega_1 = \frac{B\omega}{2} = \frac{R}{2L} \Rightarrow \omega_1 = \omega_0 - \frac{B\omega}{2}$

$\omega_2 - \omega_0 = \frac{B\omega}{2} = \frac{R}{2L} \Rightarrow \omega_2 = \omega_0 + \frac{B\omega}{2}$

Selectivity:

$$S \propto \frac{1}{B.\omega}$$

S: Resonance frequency / Band width.

$S = \frac{f_0}{f_2 - f_1} = \frac{\omega_0}{\omega_2 - \omega_1}$

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$$S = \frac{f_0}{\frac{R}{2\pi L}} = \frac{2\pi f_0 L}{R} = \frac{\omega_0 L}{R} = Q_0$$

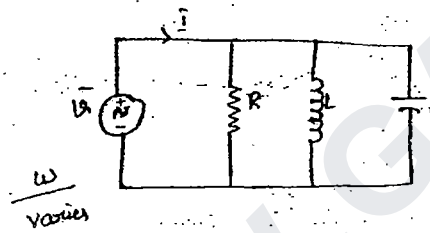
→ Selectivity is Quality Factor at Resonance.

Parallel Resonance / Anti Resonance:

during parallel Resonance in ckts the net Susceptance of the Ckt is Zero

↔ The Supply voltage and Current are in phase so ckt is purely Resistive hence pf is '1'

Case ii: General parallel RLC Ckt



$$Y_R = \frac{1}{R}$$

$$Y_L = \frac{1}{jX_L} = \frac{-j}{X_L}$$

$$Y_C = \frac{1}{-jX_C} = \frac{j}{X_C}$$

$$Y_i = Y_R + Y_L + Y_C$$

$$Y_i = \frac{1}{R} - \frac{j}{X_L} + \frac{j}{X_C}$$

$$Y_i = \frac{1}{R} + j\left[\frac{1}{X_C} - \frac{1}{X_L}\right]$$

during Resonance $\omega = \omega_0$ net Susceptance is Zero

$$\frac{1}{X_C} - \frac{1}{X_L} = 0$$

$$X_L = X_C$$

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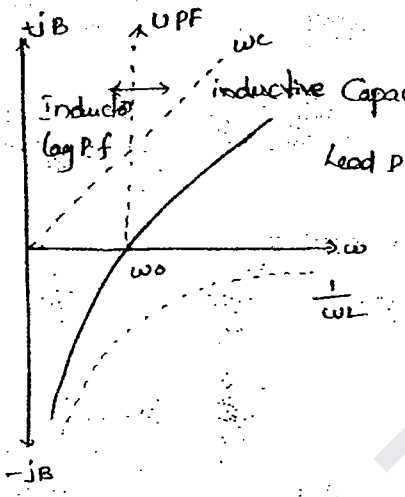
$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0^2 = \frac{1}{LC} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

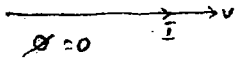
Susceptance diagram:-

$$Y = \frac{1}{R} + \left[\omega C - \frac{1}{\omega L}\right]$$

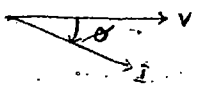


Phasor diagram

i) $\omega = \omega_0 \Rightarrow Y = \frac{1}{R}$

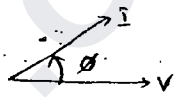


ii) $\omega < \omega_0 \Rightarrow Y = \frac{1}{R} - jB$



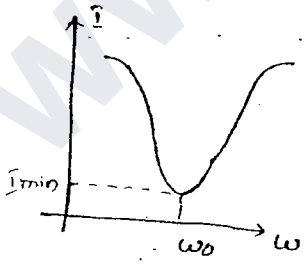
Ø = -ve

iii) $\omega > \omega_0 \Rightarrow Y = \frac{1}{R} + jB$

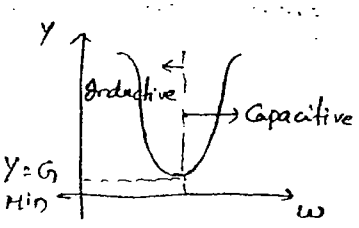


Ø + is +ve

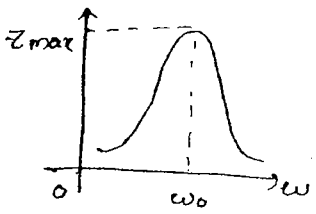
I (vs) ω :-



Y (vs) ω :-



Z (vs) ω :



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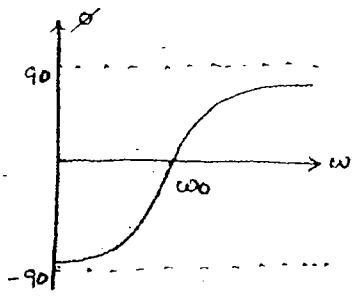
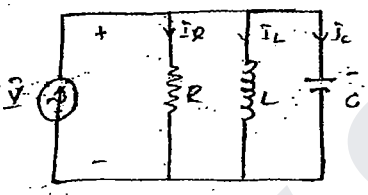
d)

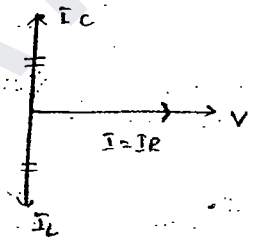


→ during parallel Resonance the net admittance is Minm & impedance is Maxm So it is called Rejector ckt.

→ during parallel Resonance the total Source Current flows through Resistance only so we Say parallel Resonant ckt as Current amplification ckt

Complete phasor diagram:





Q Factor at Resonance

$$\because \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q_0 = \frac{R}{\omega_0 L} = \omega_0 RC$$

$$Q_0 = R\sqrt{\frac{C}{L}}.$$

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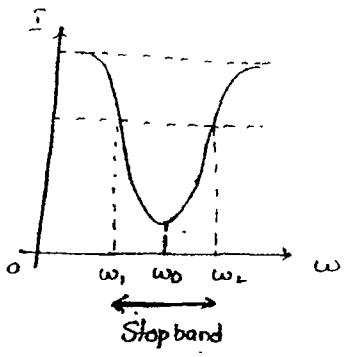
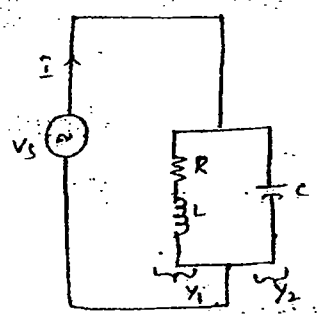


→ Same as

practical parallel Resonance ckt [Tank ckt]



$$Y_1 = \frac{1}{Z_1} = \frac{1}{R+jX_L} = \frac{R-jX_L}{(R+jX_L)(R-jX_L)}$$

$$= R-jX_L \,/\, R^2+X_L^2$$

$$Y_2 = \frac{1}{Z} = \frac{1}{-jX_C} = j/X_C$$

$$Y_T = Y_1+Y_2 = \frac{R}{R^2+X_L^2} + j\left[\frac{1}{X_C} - \frac{X_L}{R^2+X_L^2}\right]$$

at Resonance

$\omega = \omega_0$ → net Susceptance = 0

$$R^2+X_L^2 = X_L$$

$$R^2+\omega_0^2 L^2 = X_L$$

$$\omega_0^2 L^2 = \frac{L}{C} - R^2$$

$$\omega_0^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \text{ rad/sec}$$

$$\omega_0 = f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \text{ Hz}$$

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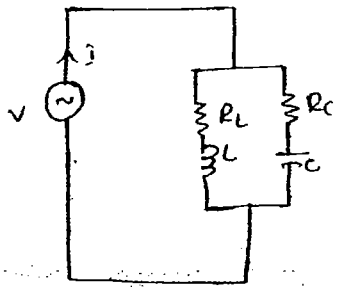
The Resonant frequency form



$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}}$$

dynamic Resistance under Resonance:

a. Series RLC

$Z = R + j[X_L - X_C]$

at $\omega = \omega_0 \Rightarrow X_L = X_C$

net Susceptance = 0 $\quad Z = R$

dynamic impedance : R.R

b, General parallel R-L-C

$Y_T = \frac{1}{R} + j\left[\frac{1}{X_C} - \frac{1}{X_L}\right]$

$\omega = \omega_0$ net Susceptance = 0

dynamic impedance $Z = 1/R$

c. Tank ckt:

$$Y_T = Y_1 + Y_2 = \frac{1}{R + jX_L} + \frac{1}{-jX_C} = \frac{R - jX_L}{R^2 + X_L^2} + \frac{j}{X_C}$$

$$Y_T = \frac{R}{R^2 + X_L^2} + j\left[\frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2}\right]$$

under resonance = 0

dynamic impedance $= Z = 1/Y = \frac{R^2 + X_L^2}{R}$

But $R^2 + X_L^2 = X_L \cdot X_C = \frac{1}{\omega C} \cdot \omega L = \frac{L}{C}$

$$Z_D = \frac{L}{RC}$$

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10/5/12

Magnetic Circuits.

charges at rest produce only electrostatic field.

Charges in motion ie Current produce both magnetic & electric fie

Comparision b/w electric and magnetic Quantities:

Electric ckt	Magnetic ckt
1) Voltage is the Cause / EMF Symbol (V) units volts	1) MMF - magneto motive force is the Cause Symbol mmf units - Amphere turns MMF = NI
2) Current is the effect Symbol (I) units Amps	2) Flux is the effect Symbol (ϕ) units - webers (wb)
3) $V \propto I$ $V = IR$ $R = \frac{V}{I} = \frac{\text{Voltage}}{\text{Current}}$ (Ω)	3. MMF $\propto \phi$ MMF $\propto \phi S$ S is called Reluctance $S = \frac{MMF}{\phi} = \frac{NI}{\phi}$ (unit $\frac{A-T}{wb}$)
4) $R = \frac{\rho l}{a}$	4) $S = \frac{l}{\mu a}$
5) Electric field intensity (EFI) $E = \frac{V}{d}$ units $\frac{volt}{m}$	5) Magnetic field intensity (H) $H = \frac{MMF}{l} = \frac{NI}{l}$ $\frac{A-T}{m}$
6) Electric Current density $J = \frac{I}{a}$ units (Amp/m^2)	6) Magnetic flux density $B = \phi/a$ units $\frac{wb}{m^2}$ or Tesla
7) $D = \epsilon E$ D - electric flux density	7) $B = \mu H$

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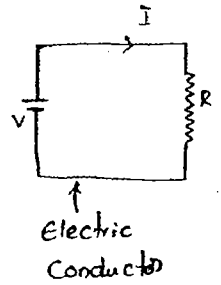
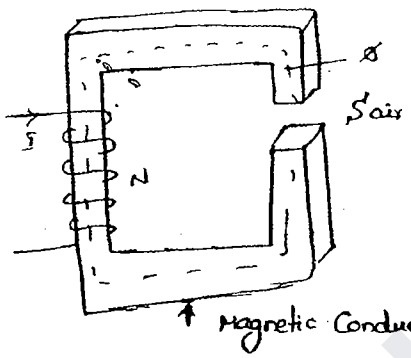


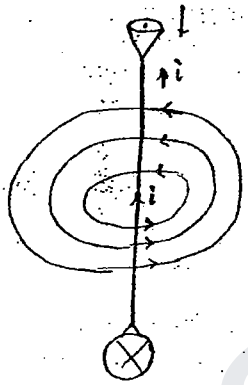


Fe, Si-fe, Cast iron

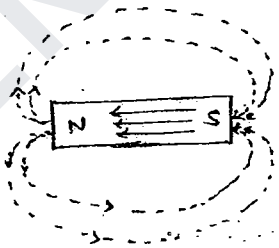
Conductance = $\frac{1}{\text{Resistance}}$

permeance = $\frac{1}{\text{Reluctance}}$

→ Ampheres Thumb rule:



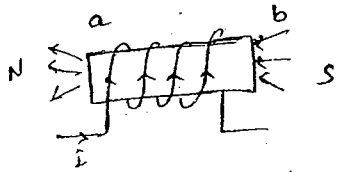
permanent magnet



Electromagnet:

a = South pole

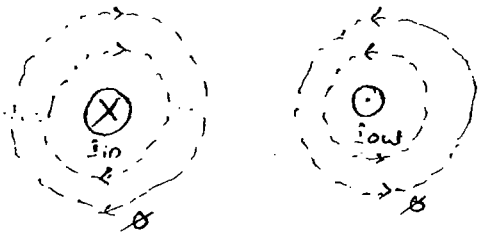
b = north pole



a - Northpole

b - South pole

Cross & dot product



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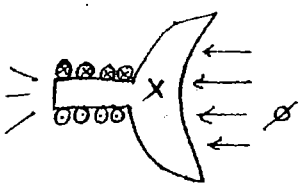
Machines



X - South pole

Closed Loop Current

X : South pole Y - North pole

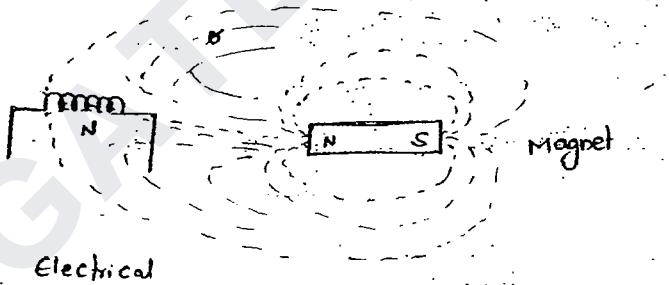
Faraday's law of Electromagnetic Induction:

→ They should be change in Flux linkage in a coil inorder to induce EMF in it.

→ The magnitude of this induced EMF ∝ Rate of change of Flux
↑ propotional.

Flux linkage

$\psi = N\phi$



Mathematically Faraday's law given by $e = -N\frac{d\phi}{dt}$

e → emf in the coil

$\frac{d\phi}{dt}$ ← rate of Change of Flux

N → no of turns of coil

(-) → due to Lenz's law (- ve)

$e = -N\left[\frac{d}{dt}\right]$

$e = -N\left[\frac{\phi_2 - \phi_1}{\Delta t}\right]$

ϕ_1 - initial flux

ϕ_2 - final flux

Δt - time difference

$e = -N\left[\frac{\phi_2 - \phi_1}{\Delta t}\right]$

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- The Emf induced in the coil by Relative movement & Change in position b/w the Electrical Conductor and magnetic medium then we Say it as dynamically induced Emf

Ex: All Rotating machines obey's faradays law dynamically

- if emf induced in the coil is by without any relative movement or Change in position b/w Electrical Conductor System & magnetic field System we Say it as Statically induced Emf

Ex: Transformers obeys ~~faraday's~~ law statically

- The Basic requirement to induce Emf statically is field Excitation Should be A.C

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Principle of Generator:

$e = Blu \sin\theta$

B - wb/m^2

l - length of conductor

u - relative velocity b/w conductor & Magnetic flux [m/sec]

θ - angle b/w $\bar{B}$ & $\bar{u}$

But in Machines $\theta = 90$

$e = Blu$

Cylindrical Machine - $\boxed{e = Blu}$

A conductor placed in a magnetic field is moved there is induced EMF unit.

principle of motor:

$F = Bil \sin\theta$

B - wb/m^2

i - Current through conductor (A)

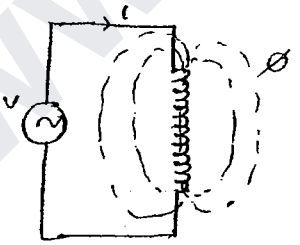
l - length of Conductor

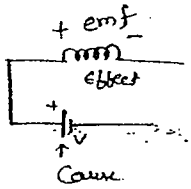
θ - angle b/w $\bar{B}$ & $\bar{I}$

F - Force developed by Conductor

A Current Carrying Conductor placed in a Magnetic field Experiences force.

Concept of Self Inductance & Self Induced EMF:





Lenz's law

when a coil excited by time varying Current produce time varying flux which is linking itself So acc to Faradays law of E.M.I the change in flux linkage with the coil induce emf in it.

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This Emf induced in the Coil because of its own Current called Self induced Emf which results into Self Inductance.

$$i = i \times \frac{\phi}{\phi} \qquad \phi = \phi \times \frac{i}{i}$$

$$i = \cdot \qquad \frac{d\phi}{dt} = \frac{\phi}{i} \cdot \frac{di}{dt}$$

But $e = -N \frac{d\phi}{dt}$

$$= -N \frac{\phi}{i} \frac{di}{dt}$$

$$\boxed{e = -L \frac{di}{dt}} \longrightarrow \text{Self induced Emf}$$

$$L = \frac{N\phi}{i} = \frac{\psi}{i} - \text{Self inductance (Henry)}$$

$$\therefore \phi = \frac{MMF}{S} = \frac{N \cdot i}{S}$$

$$\phi = \frac{N \cdot \left[\frac{N\phi}{L}\right]}{S} = \frac{N^2 \phi}{LS} \Rightarrow \boxed{L = \frac{N^2}{S}}$$

But $S = \frac{l}{\mu a}$

$$L = \frac{N^2}{l/\mu a} \Rightarrow \boxed{L = \frac{\mu N^2 a}{l}}$$

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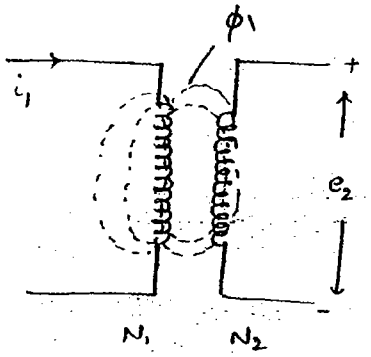
Concept of Mutual induced Emf and Mutual inductance:-

The voltage E_2 induced in 2nd coil because of 1st coil current hence called Mutually induced EMF



$$\phi_1 = \phi_{11} + \phi_{12}$$

↑ Flux linking Both coils.

which Exhibit Mutual Inductance.

Acc to Faraday's law

$$e_2 = -N_2 \frac{d\phi_{12}}{dt}$$

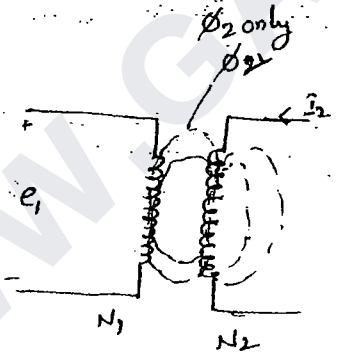
$$\phi_{12} = \frac{\phi_{12}}{i_1} \times i_1$$

$$\phi_{12} = \frac{\phi_{12}}{i_1} \cdot i_1$$

$$\frac{d\phi_{12}}{dt} = \frac{\phi_{12}}{i_1} \frac{di_1}{dt} \Rightarrow e_2 = -N_2 \frac{\phi_{12}}{i_1} \frac{di_1}{dt}$$

$$e_2 = -\left[\frac{N_2 \phi_{12}}{i_1}\right] \frac{di_1}{dt}$$

$$e_2 = -M \frac{di_1}{dt}$$ ← Mutual Inductance



$$e_1 = -\left[\frac{N_1 \phi_{21}}{i_2}\right] \frac{di_2}{dt}$$

$$M = \frac{N_1 \phi_{21}}{i_2}$$

$$M = \frac{N_1 \phi_{21}}{i_1} = \frac{N_2 \phi_{12}}{i_2}$$

$$M = \frac{N_1 \phi_C}{i_1} = \frac{N_2 \phi_C}{i_2}$$

But practically $\boxed{\phi_{12} = \phi_{21} = \phi_C}$

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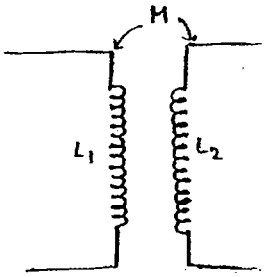
→ System of two coils:



$M \leq \sqrt{L_1 L_2}$

K is always ≤ 1

$M = K\sqrt{L_1 L_2}$

$K = \frac{M}{\sqrt{L_1 L_2}}$

$K \in [0, 1]$

K - Coefficient Coupling

if $K = 0$ we say coils are loosely coupled

Ex: air cored coils used in communication

if $K = 1$ we say the coils are tightly coupled

Ex: Transformers

Energy Stored Coupled Coils

$$E = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 \pm M i_1 i_2 \quad \text{joules}$$

+ ⟶ if Mutually adding fluxes

− ⟶ if Mutually opposing fluxes

→ Mutual inductance is a +ve quantity but mutual emf can be +ve & −ve

→ determining the correct polarity of this Mutual voltage is not easy hence we use dot Convention

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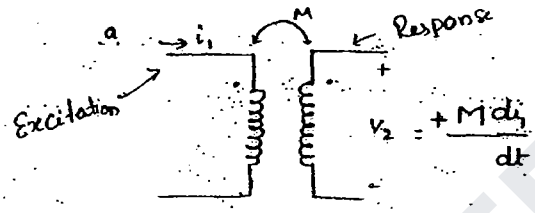
dot Convension:

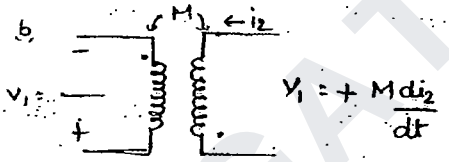
Statement-1:

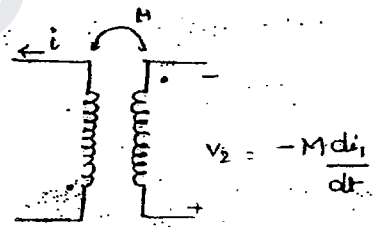
if Current Enters the dotted terminal of 1st coil then the polarity of Mutual voltage is +ve at the dotted terminal of 2nd coil

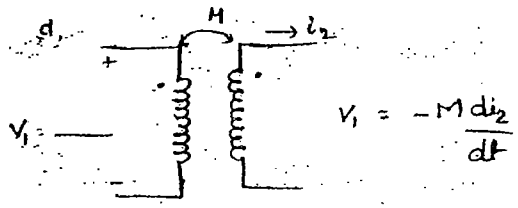
Statement 2 if Current Leaves the dotted terminal of 1st coil then the polarity of Mutual voltage is -ve at the dotted terminal of 2nd coil

Ex: Identify the Correct polarity of Mutual voltages in the System of coils below.



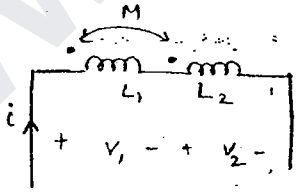






Series Connection of coils [with mutual Inductance]

a) Series aiding



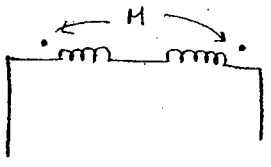
$L_{eq} = L_1 + L_2 + 2M$

$V_1 = V_S + V_m = L_1\frac{di}{dt} + M\frac{di}{dt}$

$V_2 = V_S + V_m = L_2\frac{di}{dt} + M\frac{di}{dt}$

$V = (L_1 + L_2 + 2M)\frac{di}{dt}$

b) Series opposing



$L_{eq} = L_1 + L_2 - 2M$

$V_1 = V_S + V_m = L\frac{di}{dt} - M\frac{di}{dt}$

$V_2 = V_S + V_m = L_2\frac{di}{dt} - M\frac{di}{dt}$

$V_1 + V_2 = (L_1 + L_2 - 2M)\frac{di}{dt}$

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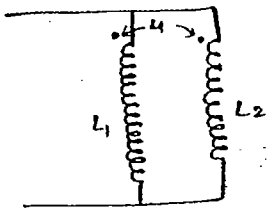
20/5/12

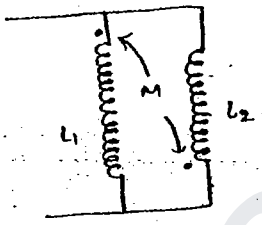
parallel Connection of coils [with mutual Induction]

parallel aiding



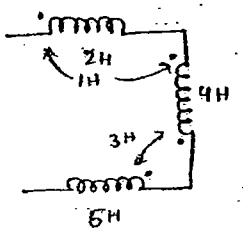
$$L_{eq} = \frac{L_1L_2 - M^2}{L_1+L_2-2M}$$

Parallel opposing



$$L_{eq} = \frac{L_1L_2 - M^2}{L_1+L_2+2M}$$

(P)1

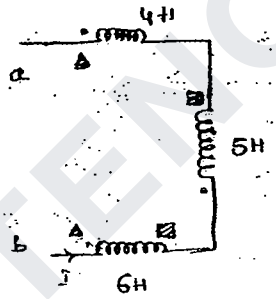


2H - 2+1 - 3

4H - 4+1-3 - 2

5H - 5 - 3 - 2 / 7

(P)



• 3H

▨ 2H •

▲ 1H

11+2
+6
13

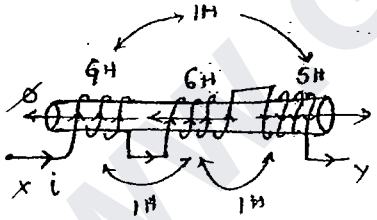
→ 4-3-1 = 0

→ 5-3+2 = 4

→ 6-1+2 = 7

11

(P3)



4+1-1 = 4

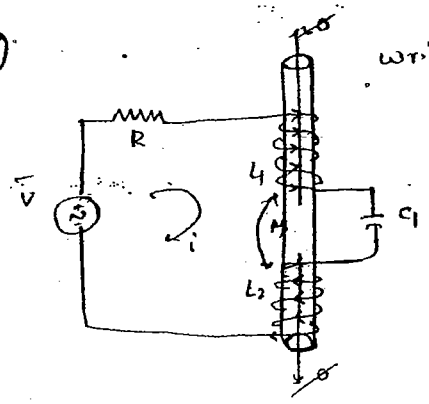
6+1-1 = 6

5-1-1 = 3

13 H

(P)

write mesh Equation



$$-V + iR + L_1\frac{di}{dt} + \frac{1}{C}\int i\,dt + L_2\frac{di}{dt} - 2M\frac{di}{dt} = 0$$

$$-\bar{V} + \bar{I}R + j\omega L_1\bar{I} - \frac{j}{\omega C}\bar{I} + j\omega L_2\bar{I} - j\omega 2M\bar{I} = 0$$

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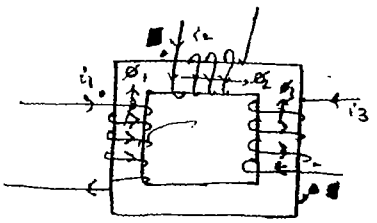
(P) place proper dot convention for 3 coils

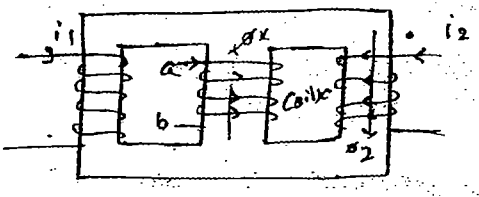


ϕ_1 – clockwise

ϕ_2 : "

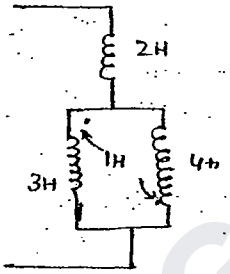
ϕ_3 – anticlockwise

(P)



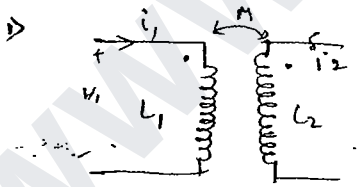
w.r.to coil 2 the proper dot for coil x at a or b

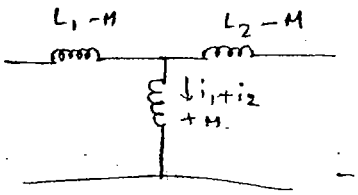
(P)



$$L_{eq} = \frac{3\times4-(1)^2}{3+4+2} = \frac{29}{9}\ H$$

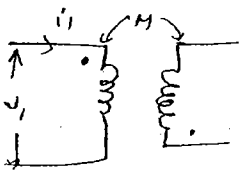
Transformer Concept:-

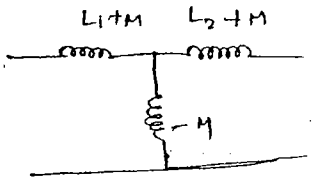




$$-V_1 + (L_1 - M)\frac{di_1}{dt} + M\frac{d}{dt}(i_1+i_2) = 0$$

$$-V + L_1\frac{di_1}{dt} - M\frac{di_1}{dt} + M\frac{di_1}{dt} + M\frac{di_2}{dt} = 0$$





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Reflection Coefficient:

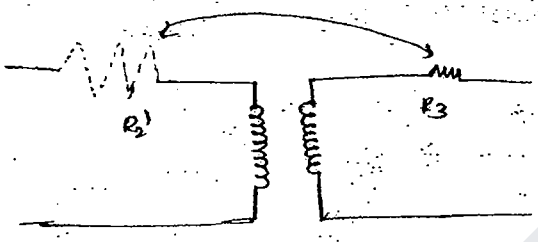
k - Transformation Ratio Voltage Transfer Ratio Transformation Ratio

$$k = \frac{N_2}{N_1} = \frac{E_2}{E_1} \approx \frac{V_2}{V_1} \approx \frac{I_1}{I_2}$$

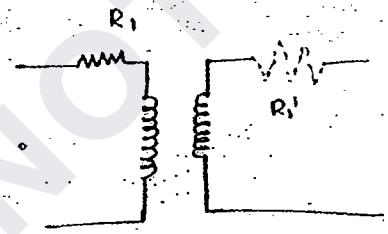
→ a - Turns Ratio

$$a = \frac{1}{k}$$

$$a = \frac{N_1}{N_2} = \frac{E_1}{E_2} \text{ or } \frac{V_1}{V_2} \approx \frac{I_2}{I_1}$$

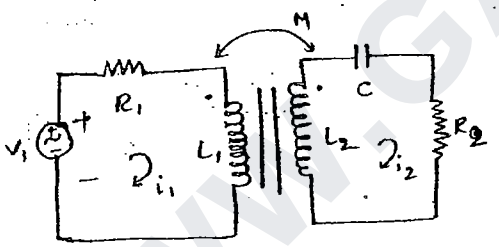


$$R_2' = \frac{R_2}{k^2}$$



$$R_1' = k^2 R_1$$

Write Mesh Equation.



$$-V_1 + i_1 R_1 + L_1 \frac{di}{dt} - M \frac{di_2}{dt} = 0$$

$$-V_1 + \bar{I}_1 R_1 + j\omega L_1 \bar{I}_1 - j\omega M \bar{I}_2 = 0$$

$$V_1 = \bar{I}_1 [R_1 + jX_1] - jX_m \bar{I}_2 = 0$$

$$L_2 \frac{di_2}{dt} + \frac{1}{C} \int i_2 dt + i_2 R_2 - M \frac{di_1}{dt} = 0$$

$$j\omega L_2 \bar{I}_2 - \frac{j}{\omega C} \bar{I}_2 + \bar{I}_2 R_2 - j\omega M \bar{I}_1 = 0$$

$$\bar{I}_2 [R_2 + jX_2 - jX_C] - jX_m \bar{I}_1 = 0$$

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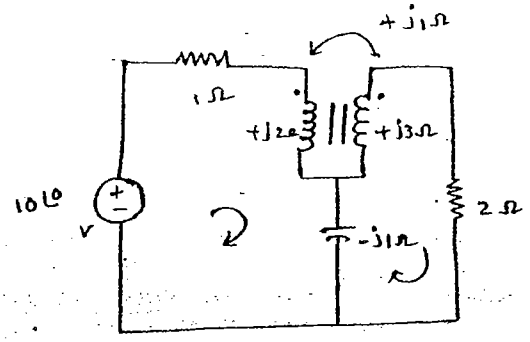
(P) Find Plost in 2Ω Res.stor



$L\frac{di}{dt}$

$$-10\angle 0 + \bar{I}_1 + j2\bar{I}_1 - j1(\bar{I}_1 - \bar{I}_2) - j1\,\bar{I}_2 = 0$$

$$(1+j1)\bar{I}_1 = 10 \quad \text{——①}$$

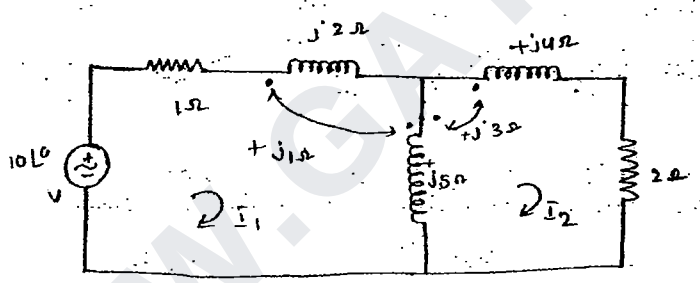
$$-j1(\bar{I}_2 - \bar{I}_1) + (2+j3)\bar{I}_2 - j1\,\bar{I}_1 = 0$$

$$(2+j2)\bar{I}_2 = 0$$

power lost = 0

$$\bar{I}_2 = 0$$

(P)



write Mesh Equations

$$-10\angle 0 + \bar{I}_1 + j1(\bar{I}_1 - \bar{I}_2) + j5(\bar{I}_1 - \bar{I}_2) + j1\,\bar{I}_1 = 0$$

$$-10\angle 0 + (1+j2)\bar{I}_1 + j5[\bar{I}_1 - \bar{I}_2] + j1[\bar{I}_1 - \bar{I}_2] + j1[\bar{I}_1] + j3[\bar{I}_2] = 0$$

$$j5[\bar{I}_2 - \bar{I}_1] + (2+j4)\bar{I}_2 - j1[\bar{I}_1] - j3[\bar{I}_2] - j3[\bar{I}_2 - \bar{I}_1] = 0$$

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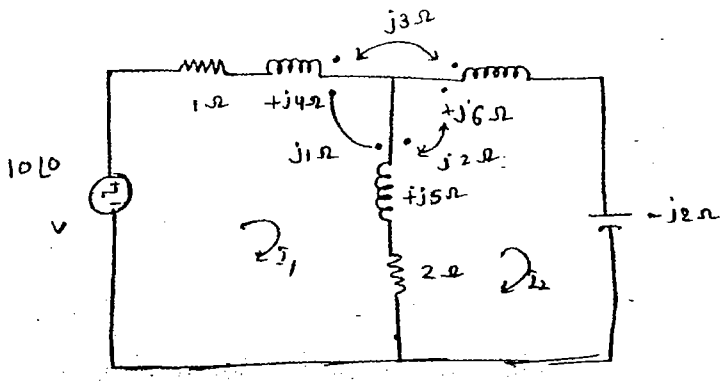
(P)



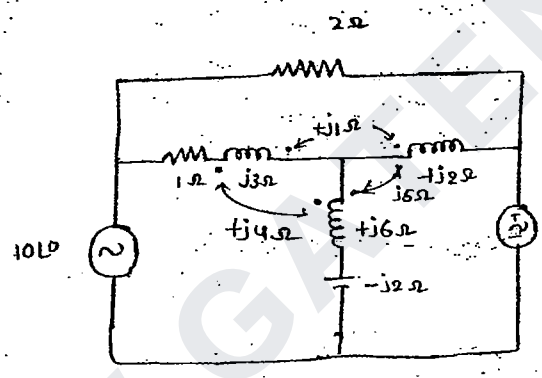
$$-10\angle 0 + (1+j4)\,\bar{I}_1 + (2+j5)[\bar{I}_1 - \bar{I}_2] - j3[\bar{I}_1 - \bar{I}_2] - j1[\bar{I}_1] + j2[\bar{I}_2] = 0$$

$$-j1[\bar{I}_1 - \bar{I}_2] - j3[\bar{I}_2] - j1[\bar{I}_1] + j2[\bar{I}_2] = 0$$

$$[2+j5][\bar{I}_2 - \bar{I}_1] + (j6 - j2)\,\bar{I}_2 + j1[\bar{I}_1] - j2[\bar{I}_2] - j3[\bar{I}_1] - j2[\bar{I}_2 - \bar{I}_1] = 0$$

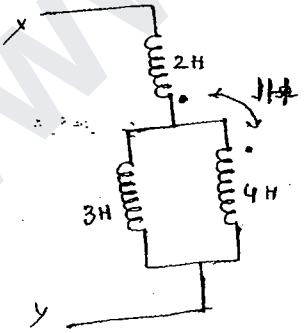
(P)

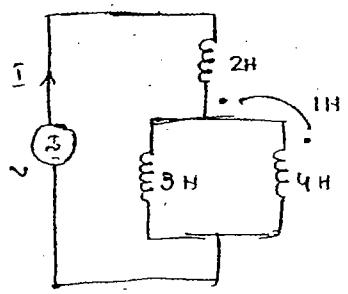
(a)



power across 2Ω

(P)





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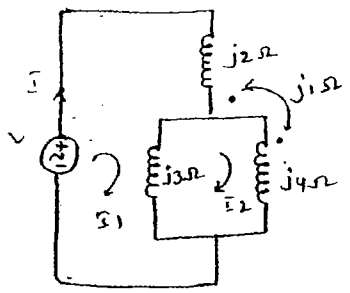


$$-V + j2\bar{I}_1 + j3(\bar{I}_1 - \bar{I}_2) - j1[\bar{I}_2] = 0$$

$$V = j5\bar{I}_1 - j4\bar{I}_2 \quad \text{—— (1)}$$

$$j3[\bar{I}_2 - \bar{I}_1] + j4\bar{I}_2 - j1[\bar{I}_1] = 0$$

$$-4\bar{I}_1 + 7\bar{I}_2 = 0$$

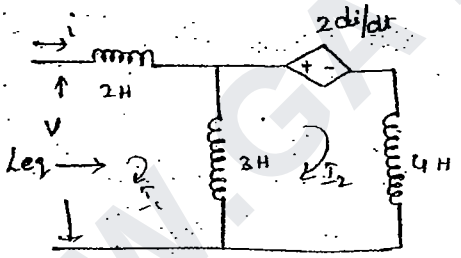
$$7\bar{I}_2 = 4\bar{I}_1$$

$$\bar{I}_2 = \frac{4}{7}\bar{I}_1$$

$$V = j5\bar{I}_1 - j4\left(\frac{4}{7}\right)\bar{I}_1$$

$$V = j\bar{I}_1\left[5 - \frac{16}{7}\right] = j\bar{I}_1\left[\frac{19}{7}\right]$$

$$\frac{V}{\bar{I}_1}$$



$$-V + 2\frac{di_1}{dt} + 3\frac{di_1}{dt} - 3\frac{di_2}{dt} = 0$$

$$5\frac{di_1}{dt} - 3\frac{di_2}{dt} = V \quad \text{—— (1)}$$

$$\frac{3di_2}{dt} - 3\frac{di_1}{dt} + 2\frac{di}{dt} + 4\frac{di_2}{dt} = 0$$

$$\frac{di}{dt} = \frac{di_1}{dt} \quad \text{—— (2)}$$

$$-\frac{di_1}{dt} + 7\frac{di_2}{dt} = 0 \quad \text{—— (3)}$$

$$5x - 3y = V \quad \Rightarrow \quad 35x - 21y = 7V$$

$$-x + 7y = 0 \qquad -3x + 21y = 0$$

$$32x = 7$$

$$x = 7/32$$

$$\frac{di_1}{dt} = \frac{7}{32}V$$

$$V = \frac{32}{7}\frac{di_1}{dt} \qquad Leq = 32/7$$

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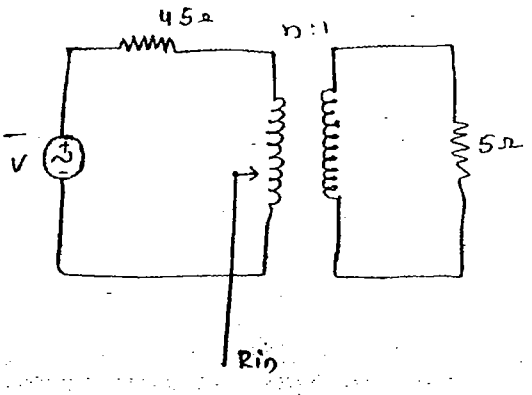
(P)



what is the value of 'n' for which power occur in 5Ω

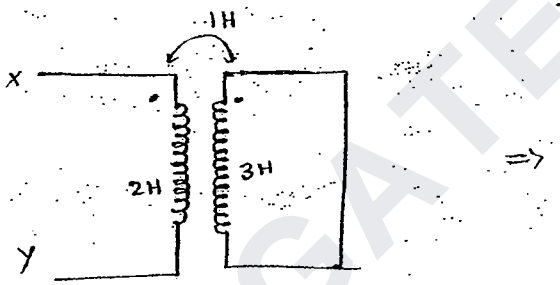
$$R_{in} = \frac{5}{k^2} = \frac{5}{(1/n)^2} = n^2 5$$

$$R_{in} = n^2 5 = R_S = 45$$

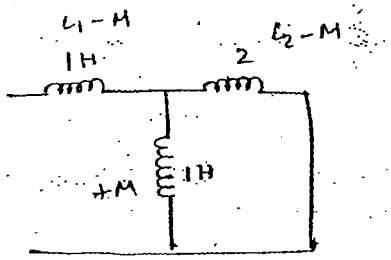
$$n^2 5 = 45$$

$$n^2 = 9 \qquad n = 3$$

(P)



$\Rightarrow$



$$1 + [2 \| 1] = 1 + \frac{2}{3} = \frac{5}{3}$$

(P) If a coil has 500 turns & linked with a flux of 50mWb while carrying a current of 125A. Calculate inductance of coil & this current is reduce to zero uniformly in 0.1sec Calc the self induced Emf of coil

$$\psi = L I$$

$$L = \frac{\psi}{I} = \frac{N\phi}{I} = \frac{500 \times 50 \times 10^{-3}}{125} = 0.2\,H$$

Self induced emf $\quad V = -L\frac{di}{dt} = -0.2\left[\frac{i_2 - i_1}{\Delta i}\right]$

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$$V = -0.2\left[\frac{0-125}{0.1}\right]$$

$$= 250V$$

(P) Two coils of Self Inductances 2H & 4H are Coupled Such that Mutual Inductance is 1.5H if mutual voltage is +ve find the amount of energy stored after 0.2 sec. if the ckt is Connected to a DC voltage Source of 12V.

$L_1 = 2H \quad L_2 = 4H \quad M = 1.5H$

$$L_{eq} = L_1 + L_2 + 2M = 2+4+2\times1.5 = 9H$$

$$V = L\frac{di}{dt} = \frac{1}{2}L_{eq}i^2$$

$$12 = 9\frac{di}{dt}$$

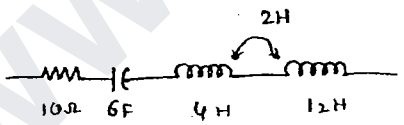
$$\int di = \frac{12}{9}\int dt \Rightarrow i = \frac{12}{9}\times 0.2$$

$$i = \frac{24}{3}\,0.266$$

$$E = \frac{1}{2}L_{eq}i^2$$

$$= \frac{1}{2}(9)(0.266)^2 = 0.32J$$

(P)



what is Resonance frequency : $\omega_0 = \frac{1}{\sqrt{LC}}$

$$L_{eq} = 4+12-4 = 12$$

$$C = 6F$$

$$\omega_0 = \frac{1}{\sqrt{12\times6}} = 1/\sqrt{72}\ rad/sec$$

$$f_0 = \frac{1}{2\pi\sqrt{72}}\ rad/sec.$$

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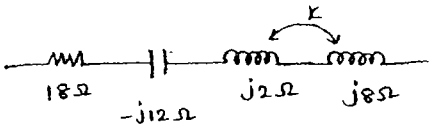
(P)



for Series Resonance to happen, k = ——

$X_L = X_C$

$\omega L_{eq} = \frac{1}{\omega C}$

$L_{eq} = L_1 + L_2 \pm 2M$

$L_{eq} = \omega j \pm 2k\sqrt{L_1 L_2}$

$\omega L_{eq} = \omega L_1 + \omega L_2 \pm 2\omega M$

$\omega L_{eq} = \omega j\omega \pm 2\omega k\sqrt{L_1 L_2}$

$M = k\sqrt{L_1 L_2}$

$\omega L_{eq} = 2 + 8 \pm \omega 2M = \frac{1}{\omega C} = j2$

$\pm \omega 2M = 2$

$\omega M = \pm 1$

$M = k\sqrt{L_1 L_2} = k\sqrt{(\omega L_1)(\omega L_2)}$

$\omega M = k\sqrt{2 \times 8} \Rightarrow 1 = k\sqrt{16}$

$k = 1/4 = 0.25$

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Electrical Transients

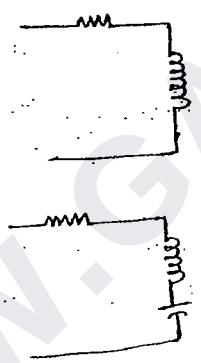
→ Voltages & Currents are our State variables in our Electrical ckt it is necessary to choose correct statevariable to observe the transient state

→ Since in inductor $V = L\frac{di}{dt}$ Current through inductor Correct State variable in RL ckt's

∴ in Capacitor $i = C\frac{dv}{dt}$ voltage across Capacitor is Correct State Variable in RC ckt's

The order of a ckt or a Network is the No of Energy Storage elements available in it.

Ex:



Series RL } 1st order

Series RLC } Second order

These voltage & Current State Variables in any ckt Exactly represented by differential Equations. Let us Consider a standard Second order differential Equation given by $\left(a_2\frac{d^2x}{dt^2} + a_1\frac{dx}{dt} + a_0x = f(t)\right)$

For this Equation

a) order 2 (∵ highest derivative

b) Time inVariant if a_0, a_1, a_2 are independent of time

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c) Linear, if function $x(t)$ is Linear i.e V & I obey ohm's law

d) Homogeneous, if forcing function $F(t)=0$

The Solution to this differential Eqn two parts

a) particular Integral which is for a particular given input. This Response depends upon type of input and its different for different types of inputs. This Response is called forced Response or) Sourced Response which is Steady state Response ($t \to \infty$)

b) Complementry function: This Response is when Source is absent & it purely depends on internal stored energy & the Nature of elements

→ This Response is always unique

Total Response = Forced response + Source free Response

↓ (Steady state Response)

↓ (Natural Response)

↓ (Transient)

Zero i/p Response o/p when i/p zero because of internal stored Energy independent to i/p But depends upon values of R, L, C

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State of a N/w (initial conditions):

It Represents those values of voltages & Currents at $t = t_0$ for which o/p can be determined for all $t \geq t_0$

Zero State Response:

This is the Response with given input ie steady state Response.

Identifying Transient problem:

a) problems with Switches

b) problems with initial condition

$V_c(0), i_L(0)$

c) problems w.r.to time V (at $t = 2$ sec)

i (at $t = \infty$)

Note:

→ An Inductor Never allows Sudden Change in Current

$$i_L(0^-) = i_L(0) = i_L(0^+)$$

→ A Capacitor Never allow Sudden change in voltage.

$$V_c(0^-) = V_c(0) = V_c(0^+)$$

$V = L\frac{di}{dt}$ } for R, L Ckt, KVL → mesh Analysis

$\because i = C\frac{dv}{dt}$ } for R-C Ckt KCL → Nodal analysis

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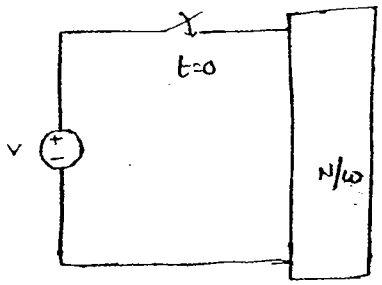

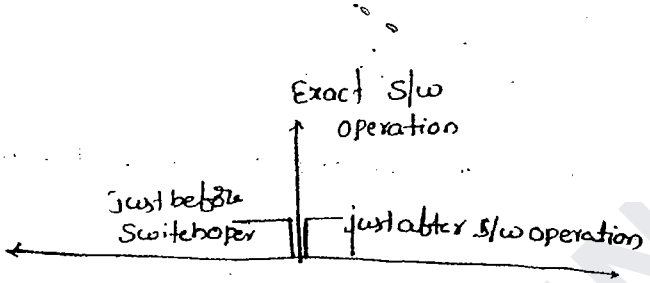

$t = 0^+ \Rightarrow +0.000000000\ldots001$

$0^- \Rightarrow -0.000000\ldots001$

$\Delta t \Rightarrow [t=0^+] - [t=0^-] = +0.000000\ldots002 \simeq 0$

So Analysis a ckt/Nw at $t=0$ means, transient analysis

Analysis at $S \to \infty$ is transient analysis.

Behaviour of passive Elements for various types of i/ps:-

	DC	AC	Transient
	$S \to 0$	$S \to$ Constat	$t = 0^+$ $S \to \infty$
R	R	R	R
L	S.C	$V_L = +jX_L I$ [Lagging] I lags V by 90°	O.C
C	O.C	$V_C = -jX_C I$ [Leading] I leads V by 90°	S.C
	$Z_R = R$ for $S \to \infty$: R	$Z_L = j\omega L = SL$ $S \to \infty \Rightarrow$ O.C	$Z_C = \frac{1}{j\omega C} = \frac{1}{SC}$ $S \to \infty$ S.C

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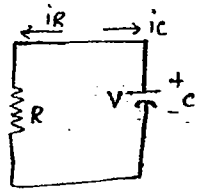
Source free Response:

1 First order circuits.

Series R-C ckt.



Let $V(0) = V_0$

KCL

$$i_R + i_C = 0$$

$$\frac{V}{R} + C\frac{dv}{dt} = 0$$

$$C\frac{dv}{dt} = -\frac{V}{R} \Rightarrow \frac{dv}{dt} =$$

$$\frac{dv}{V} = \frac{-dt}{RC}$$

$$\int \frac{dv}{v} = \int \frac{-1}{RC}\,dt$$

$$\ln[V] = \frac{-t}{RC} + \ln[A]$$

$$\ln\left[\frac{V}{A}\right] = -t/RC$$

$$V = A \cdot e^{-t/RC}$$

at $t=0$, $V = V_0$

$0 < t < 4\tau$ — Transient

$t \geq 5\tau$ — steady state

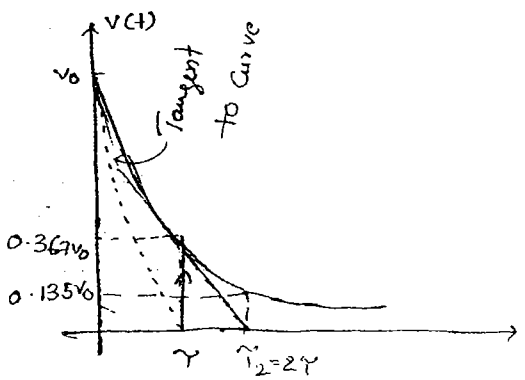
$$A = V_0$$

$$V(t) = V_0 \cdot e^{-t/RC}$$

$$V(t) = V_0 \cdot e^{-t/\tau}$$

$\tau = RC$ Time Constant for Series R-C Circuit

4τ Constants Transient $0 < t < 4\tau$	V →	$t = 0$	V_0
	V →	$t = \tau$	$0.367 V_0$
	V →	$t = 2\tau$	$0.135 V_0$
	V →	$t = 3\tau$	$0.049 V_0$
	V →	$t = 4\tau$	$0.018 V_0$
Steady state	← V →	$t = 5\tau$	$0.0067 V_0$



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Time Constant:

It is defined as time taken to Reach 36.7% of its initial value (or),

* It is also defined as time taken to Reach 63.4% to its final value.

Expression for power dissipated by Resistor.

$$P_R = \frac{V^2}{R} = \frac{(V_0 \cdot e^{-t/\tau})^2}{R} = \frac{V_0^2}{R} e^{-t/(\tau/2)}$$

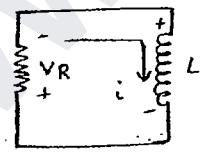
Expression for Energy in the Capacitor $\frac{1}{2} CV^2$

$$= \frac{1}{2} C (V_0 \cdot e^{-t/\tau})^2$$

$$E_C = \frac{1}{2} C V_0^2 e^{-t/\tau/2} \text{ Joules}$$

power dissipated by Resistor & Energy decay by Capacitor decays two times Faster than Voltage decay.

b) Source free RL Ckt:-



$$I_L(0) = I_0$$

KVL

$$V_R + V_L = 0$$

$$iR + L\frac{di}{dt} = 0$$

$$L\frac{di}{dt} = -Ri$$

$$\frac{di}{i} = -\frac{R}{L} dt$$

$$\int \frac{di}{i} = \int -\frac{R}{L} dt$$

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$$\ln[i] = -\frac{R}{L}t + \ln[A]$$

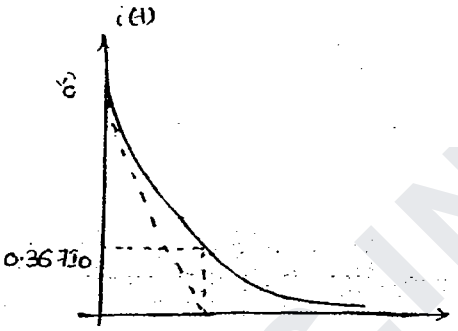
$$\ln\left[\frac{i}{A}\right] = -\frac{R}{L}t$$

$$\boxed{i = A \cdot e^{-\frac{R}{L}t}}$$

But $t=0 \quad i = I_0 \quad A = I_0$

$$** \quad \boxed{i(t) = I_0 \, e^{-t/\tau R}}$$

$$\boxed{i(t) = I_0 \cdot e^{-t/\tau}}$$



when $\tau = \frac{L}{R}$ → Time Constant of Series R-L Ckt

Expression for power in Resistor

$$P_R = I^2R = (I_0 \cdot e^{-t/\tau})^2 R = I_0^2 R \cdot e^{-t/\tau/2} \text{ Watts}$$

Energy stored in inductor $E_L = \frac{1}{2}Li^2 = \frac{1}{2}L\left[I_0 \cdot e^{-t/\tau/2}\right]^2$

$$= \frac{1}{2}L I_0^2 \, e^{-t/\tau/2} \text{ joules}$$

The power in Resistor Energy in inductor decays 2 times ~~faster~~ than Current

→ in terms of unit units of time Constant : Seconds

Sec = Sec

$\tau_V = \tau_i$

$Rc = \frac{L}{R}$

$\frac{R^2C}{L} = 1$ → unit less

$\frac{L}{R^2C} = 1$ → unit less

$R = \frac{L}{RC}$ → unit is Ω

$R^2 = \frac{L}{C}$ → unit is $(\Omega)^2$

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$R = \sqrt{\frac{L}{C}}$ — unit is Ω

$L = R^2C$ → unit is Henry

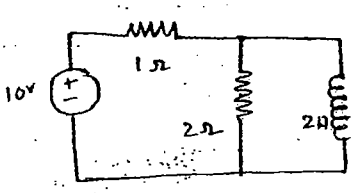
$C = \frac{L}{R^2}$ → unit is Farad

$\frac{RC}{L} = \frac{1}{R}$ → unit is ohm (υ) mho

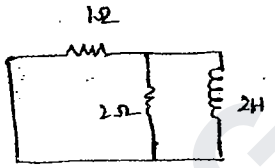
$$Z_C = \sqrt{\frac{z}{y}} = \sqrt{\frac{r+j\omega L}{g+j\omega c}} = \sqrt{\frac{L}{C}}$$

⑦ Find Time Constant of Ckts below.

(P)



$\Rightarrow \frac{L}{Req}$

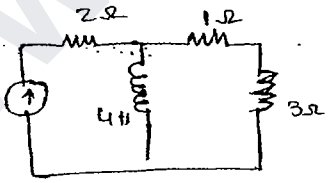


$Req = 1 \| 2 = 2/3$

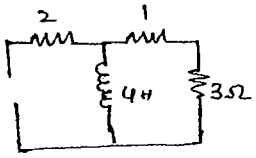
$L = 2$

$\tau = \frac{L}{Req} = \frac{2}{2/3} = 3 sec$

(P)



$\Rightarrow$



$\tau = \frac{4}{4} = 1 sec$

(P)

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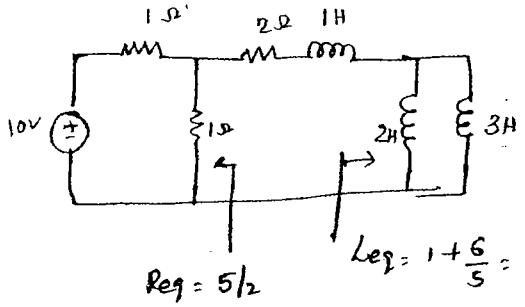
(P)

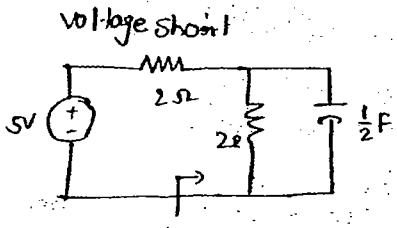


$R_{eq} = 5/2$

$L_{eq} = 1 + \frac{6}{5} = 11/5$

$\Rightarrow \tau = \frac{L_{eq}}{R_{eq}} = \frac{11/5}{5/2} = 22/25$ sec

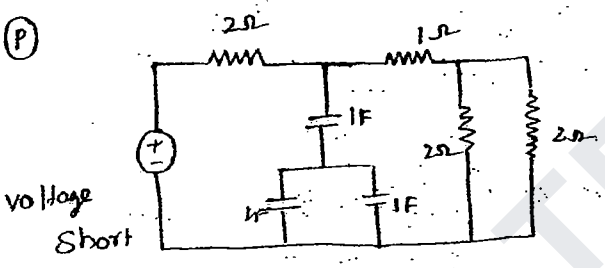
(P)



$R_{eq} = 1\Omega$

$\Rightarrow \tau = R_{eq} \cdot C = 1 \times \frac{1}{2} = 1/2$ sec

(P)



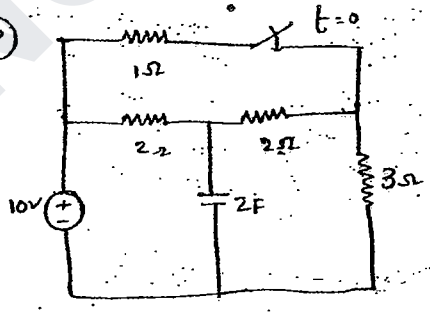
$C_{eq} = 1F$

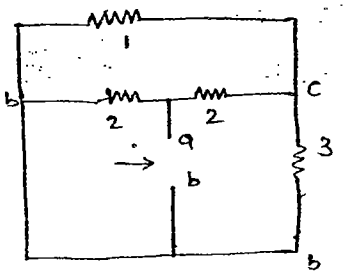
$R_{eq} = 1$

$\tau = RC$

$= 1$ Sec

(P)





$R_{eq} = 2 // [2 + [1 // 3]]$

$= 22/19$

$\tau = R_{eq} \cdot C$

$= \frac{22}{19} \times 2 = \frac{44}{19}$ Sec.

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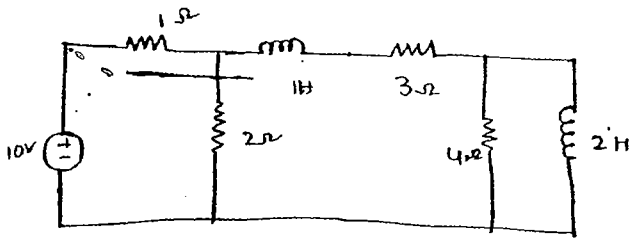
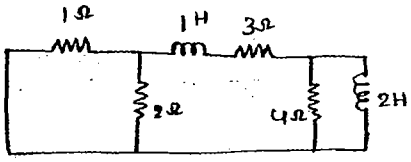
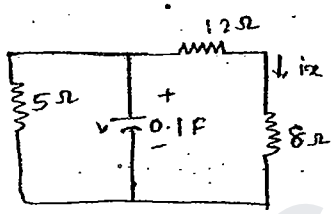
Ex:





Such ckts where R, L, C parameters ~~of~~ cannot be grouped together will have multiple time constants in multiple segments & such ckts can be Analysed only by Laplace transform method.

(P)



if $v(0) = 15$ volts

Find Complete Expression for i_x

$$v(t) = v_0 \cdot e^{-t/\tau}$$

$v_0 = 15$ volts (given)

$$\left.\begin{aligned} \tau &= R_{eq} C \\ R_{eq} &= 4\Omega \end{aligned}\right\} \quad \tau = 0.1 \times 4 = 0.4 \text{ sec}$$

$$v(t) = 15 \cdot e^{-2.5t} \text{ V} \quad \underline{t \geq 0}$$

$$i_x = \frac{v(t)}{12+8} = \frac{15 e^{-2.5t}}{20} = 0.75 e^{-2.5t} \text{ Amps}$$

$t \geq 0$

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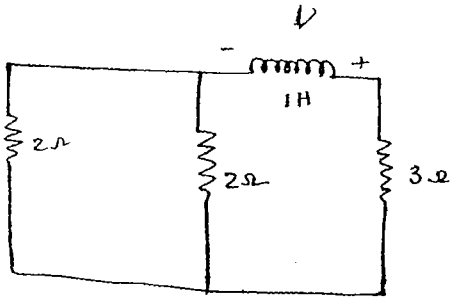
(P)



if $I(0) = 2A$

find 'V' for $t \geq 0$

$i(t) = I_0 \cdot e^{-t/\gamma}$

$I_0 \rightarrow 2A$

$\gamma = \frac{L}{Req}$; $Req = 1+3 = 4\ \Omega$

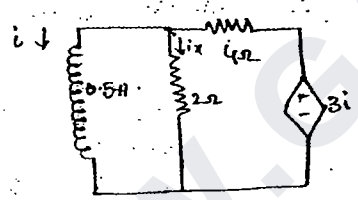
$\gamma = \frac{1}{4}$ sec

$i(t) = 2 \cdot e^{-4t}$ A

$V(t) = -L\frac{di}{dt} = -1 \frac{d}{dt}(2e^{-4t})$

$= -2 \cdot (-4)\, e^{-4t} = 8 \cdot e^{-4t}$ Volts

(P)



If $i(0) = 10A$ find (i_x) for $t \geq 0$

→ State variable is inductor ; Current.

→ Source free RL ckt

$i(t) = I_0 \cdot e^{-t/\gamma}$ form

Solution by time domain

for RL ckt KVL → mesh

$\frac{1}{2}\frac{di_1}{dt} + 2(i_1 - i_2) = 0$ —①

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$2[i_2 - i_1] + 4i_2 + 3i = 0$

$-2i_1 + 6i_2 + 3i = 0$ — (2)

1 CH (10)

$i = -i_1$ — (3)

$-2i_1 + 6i_2 - 3i_1 = 0$

$5i_1 + 6i_2 = 0 \Rightarrow i_2 = \frac{-5i_1}{6}$

i_2 is in (1) $\frac{1}{2}\frac{di}{dt} + 2i_1 - 2\left[\frac{5}{6}i_1\right] = 0$

$\frac{1}{2}\frac{di}{dt} + i_1\left(-\frac{1}{3}\right) = 0$

$\frac{di}{dt} = \frac{-2}{3}i_1$

$i_1(t) = I_1(0)\, e^{-2/3t}$

$i(0) = 10$

$I_1(0) = -10$

$i_1(t) = -10 \cdot e^{-2/3t}$

$i(t) = -\{i_1(t)\}$

$i(t) = +10 \cdot e^{-2/3t}$

(X) $\begin{cases} V_L(t) = L\frac{di(t)}{dt} = 0.5 \cdot \frac{d}{dt}\left[10\, e^{-2/3t}\right] \\ V_L = \frac{10}{2} \times -\frac{2}{3} e^{-2/3t} = -\frac{10}{3} e^{-2/3t} \text{ Volts} \end{cases}$

$i_x = \frac{V_L}{2} = -\frac{5}{3} e^{-2/3t}$ A

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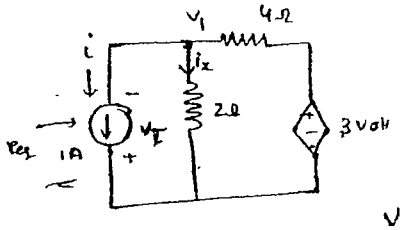
$I_0 = 10A$

$\tau = \frac{L}{R_{eq}}$ =



$R_{eq} = \frac{V_T}{I}$

$+1 + \frac{V_1}{2} + \frac{V_1 - 3}{4} = 0$

$4 + 2V_1 + V_1 - 3 = 0$

$3V_1 = -1$

$V_1 = -1/3$

$V_T = -V_1 = 1/3$ } $R_{eq} = 1/3\ \Omega$

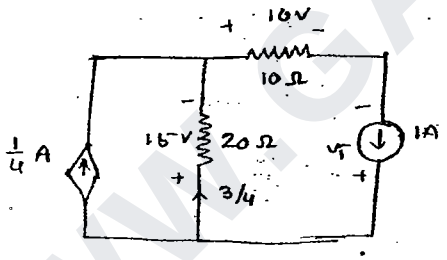
$\tau = \frac{L}{R_{eq}} = \frac{0.5}{1/3} = 3/2$

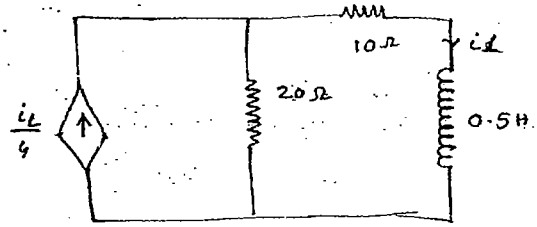
$i(t) = 10 \cdot e^{-t/3/2} = 10 \cdot e^{-2/3\,t}$

$V_L(t) = L\frac{di}{dt}$

(1) becomes

(P) $i_L(0) = 10A$, $i_L(t)$ $t \geq 0$





$+15 + 10 - V_T = 0$

$R_{eq} = \frac{V}{i} = \frac{V_T}{1A}$

$V_T = 25$ $R_{eq} = 25\Omega$ $\tau = \frac{1}{50}$

$i_L(t) = 10 \cdot e^{-50t}$

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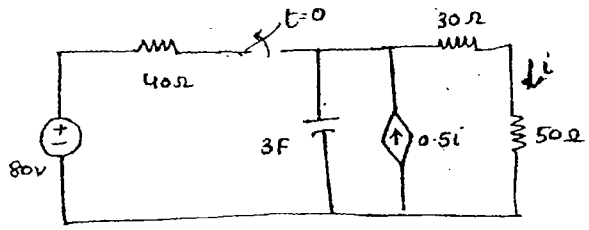
* Find $i(t)$ for $t \geq 0$.

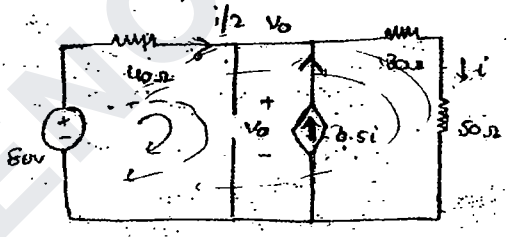


→ State Variable $V_c(t)$

∴ Switch operation indicate that Source free Response

$$V_c(t) = V_0 \cdot e^{-t/\tau}$$

→ V_0 : Ckt before Switch operation. ($t = 0^-$)



$$\frac{V_0 - 80}{40} - \frac{i}{2} + \frac{V_0}{80} = 0 \quad \text{— (1)}$$

$$i = \frac{V_0}{80} \quad \text{— (2)}$$

$$\frac{V_0 - 80}{40} - \frac{V_0}{160} + \frac{V_0}{80} = 0 \Rightarrow V_0 = 64V$$

$$-80 + 40(i/2) + 30i + 50i = 0$$

$$100i = 80$$

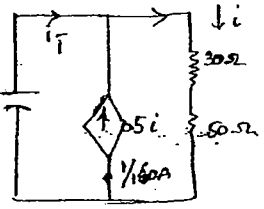
$$i = \frac{80}{100} = \frac{8}{10}$$

$$-80 + 40\left(\frac{8}{10}\right) + V_0 = 0$$

$$V_0 = 80 - 16 = 64$$

Ckt for $t \geq 0$

Capacitor Sudden change in V_0 Hoge across it



$i = \frac{1}{80}$ A

instead of Capacitor place 1V

$$i_1 = \quad 0.5i + i_1 = 1/80$$

$$i_1 = \frac{1}{80} - 0.5 \cdot \frac{1}{80}$$

$$R = \frac{1}{i_1} = \frac{1}{\frac{1}{160}} = 160\Omega$$

$$= 1/160 \text{ A}$$

$\tau = R_{eq} C$

$= 160 \times 3 = 480$ sec

$\tau = RC$

$V(t) = V_0 e^{-t/\tau}$

$V(t) = 64 \cdot e^{-t/480}$

$V(t) = 64 \cdot e^{-t/\tau}$

$i(t) = \frac{V(t)}{30+80}$

$i(t) = \frac{V(t)}{80} = \frac{64 \cdot e^{-t/480}}{80} = 0.8\, e^{-0.0028t}$

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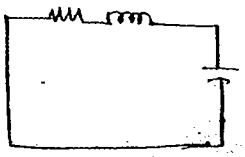
Source free 2nd order ckt:

Standard Series RLC ckt



Here two state variables allowed.

$i(0) = I_0$ (Inductor)

$v(0) = v_0 \rightarrow$ capacitor

KVL

$$iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0$$

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0 \Rightarrow \frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

using Laplace Transform

$$LS^2 I(s) + RS\,I(s) + \frac{I(s)}{C} = 0$$

$$I(s)\left[S^2 + \frac{R}{L}S + \frac{1}{LC}\right] = 0$$

$I(s) \neq 0$

$$S^2 + \frac{R}{L}S^2 \qquad S^2 + \frac{R}{L}S + \frac{1}{LC} = 0$$

$$S_1 S_2 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$$

$$S_1 S_2 \quad -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}$$

Let

$\alpha = \frac{R}{2L} \rightarrow$ Damping factor

$\omega_0 = \frac{1}{\sqrt{LC}} \rightarrow$ undamped / natural frequencies

$$S_1 S_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

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Case a:

$\alpha > \omega_0$ → over damped Case — -ve, Real & un Equal

$\frac{R}{2L} > \frac{1}{\sqrt{LC}} \Rightarrow R > 2\sqrt{\frac{L}{C}}$

The Response is

$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

Case b:

$\alpha = \omega_0$ — Critically damped — -ve, Real, Equal

$\frac{R}{2L} = \frac{1}{\sqrt{LC}} \Rightarrow R = 2\sqrt{\frac{L}{C}}$

The Response is

$i(t) = (A_1 + A_2 t) e^{-\alpha t}$

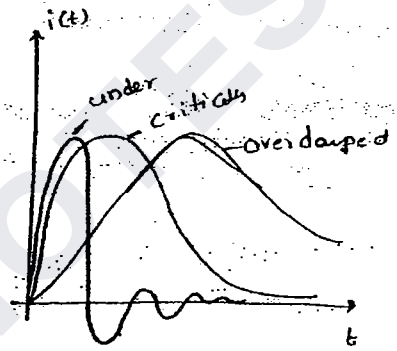
Case c:

$\alpha < \omega_0$ — under damped

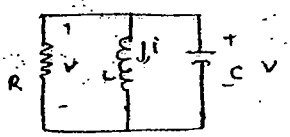
Roots are Complex Conjugate $\frac{R}{2L} < \frac{1}{\sqrt{LC}}$ $R < 2\sqrt{\frac{L}{C}}$

The Response $i(t) = e^{-\alpha t}\left[A_1 \cos\omega_d t + A_2 \sin\omega_d t\right]$

$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$



Standard parallel RLC ckt



Two State Variables $V(0)^- = V_0 \longrightarrow C$

$I(0) = I_0 \longrightarrow L$

V → State Variable

KCL:

$$\frac{V}{R} + \frac{1}{L}\int V dt + C\frac{dv}{dt} = 0$$

$$C\frac{d^2v}{dt^2} + \frac{v}{L} + \frac{1}{R}\frac{dv}{dt} = 0$$

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{v}{LC} = 0$$

$$S^2V(S) + \frac{S}{RC}V(S) + \frac{V(S)}{LC} = 0$$

$$V(S)\left[S^2 + \frac{S}{RC} + \frac{1}{LC}\right] = 0$$

$V(S) \neq 0$

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$$S^2 + \frac{S}{RC} + \frac{1}{LC} = 0$$

There are two roots

$$S_1 S_2 = \frac{-\frac{1}{RC} \pm \sqrt{\left(\frac{1}{RC}\right)^2 - \frac{4}{LC}}}{2}$$

$$= \frac{-1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}$$

Let $\alpha = \frac{1}{2RC}$ } Damping frequency

$\omega_0 = \frac{1}{\sqrt{LC}}$ } natural frequency.

Case i:

$\alpha > \omega_0$ over damped

$$\frac{1}{2RC} > \frac{1}{\sqrt{LC}} \Rightarrow R < \frac{1}{2}\sqrt{\frac{L}{C}}$$

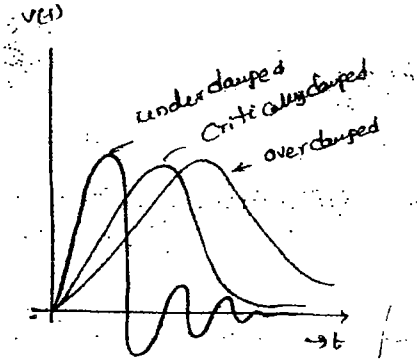
$$V(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}$$

Case ii: $\alpha = \omega_0$ Critically damped

$$\frac{1}{2RC} = \frac{1}{\sqrt{LC}} \Rightarrow R = \frac{1}{2}\sqrt{\frac{L}{C}}$$

Case iii: $\alpha < \omega_0$ under damped

$$\frac{1}{2RC} < \frac{1}{\sqrt{LC}} \Rightarrow R < \frac{1}{2}\sqrt{\frac{L}{C}}.$$



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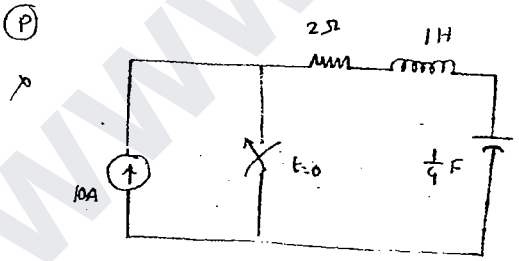

The Nature of current Exp of Current in ckt is

After Switch operation (i) is Some free Response

Series RLC

under damped

over damped $\alpha < \omega_0$

$$\alpha = \frac{R}{2L} = \frac{2}{2} = 1$$

$$\omega_0 = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{1 \times \frac{1}{9}}} = 3$$

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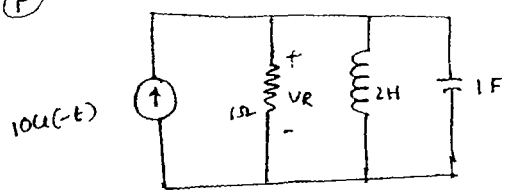
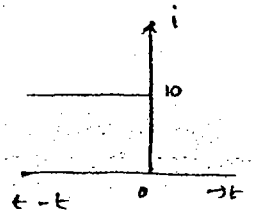
(P)



'V_R'

The voltage beyond looks like



$t>0$ does not Exist.

∴ So i is open ckt

Source free parallel RLC

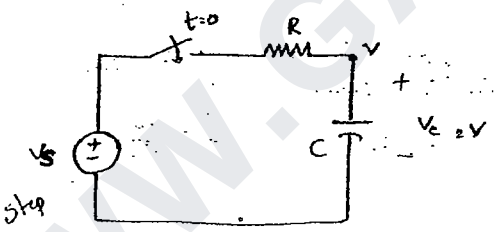
$$\alpha = \frac{1}{2RC} = \frac{1}{2\times1\times1} = 1/2 = 0.5$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2\times1}} = 0.707$$

$\alpha < \omega_0$ ... under damped

Step Response of 1st order ckt:-

a) Series R-C ckt:-



state variable is 'V_c'

KCL

$$\frac{V-V_S}{R} + C\frac{dv}{dt} = 0$$

$$V(t) = V(\alpha) + \left[V(0) - V(\alpha)\right]e^{-t/\tau}$$

$V(0)$ initial voltage across Capacitor before Switch operation

$V(\alpha)$ → final voltage across Capacitor after Switch operation

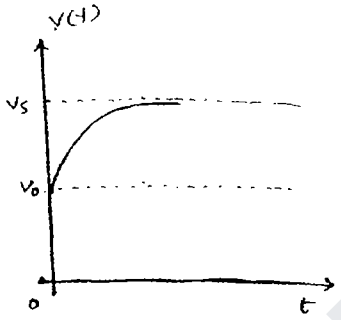
WWW.GATENOTES.IN

$\tau = RC$

(i) With initial voltage ✓

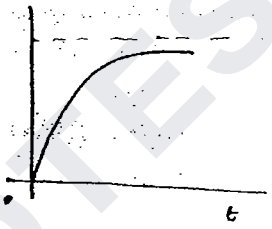
Let $V(0) = V_0$.

$$V(t) = V_s + [V_0 - V_s] e^{-t/\tau}$$



(ii) with out initial voltage $v(0) = 0$

$$v(t) = V_s [1 - e^{-t/RC}]$$



(iii) Exp for Current through Capacitor

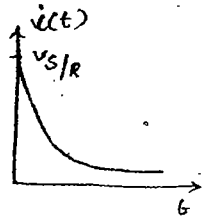
$$i = C \frac{dv}{dt}$$

$$i = C \frac{d}{dt} [V_s (1 - e^{-t/RC})]$$

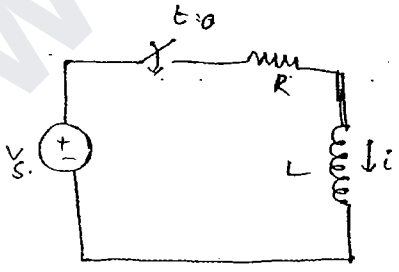
$$= C V_s [0 - e^{-t/\tau} \cdot -\frac{1}{\tau}]$$

$$i_c(t) = \frac{C V_s e^{-t/\tau}}{RC}$$

$$i_c(t) = \frac{V_s}{R} e^{-t/\tau}$$



Series R-L Ckt:



. State variable — i

$$-V_s + iR + L\frac{di}{dt} = 0$$

KVL

$$i(t) = I(\infty) + [I(0) - I(\infty)] e^{-t/\tau}$$

$I(0)$ → initial Current through Inductor before S/w operation

$I(\infty)$ — final Current through Inductor after S/w

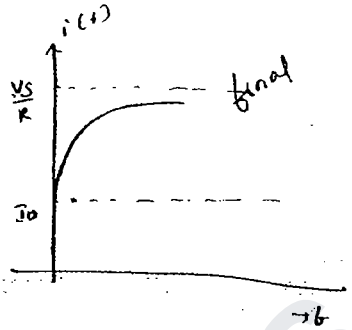
WWW.GATENOTES.IN

$\tau = L/R$

(i) with initial Current

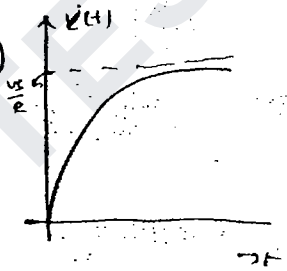
Let $i(0) = I_0$

$$i(t) = \frac{V_S}{R} + \left[I_0 - \frac{V_S}{R}\right] e^{-t/\tau}$$



(ii) with initial Current

Let $i(0) = 0$ $\quad i(t) = \frac{V_S}{R}\left(1 - e^{-t/\tau}\right)$



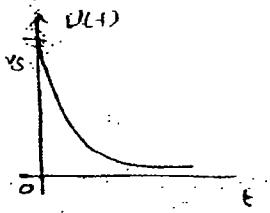
(iii) When Exp for Voltage across Inductor

$$V = L\frac{di}{dt}$$

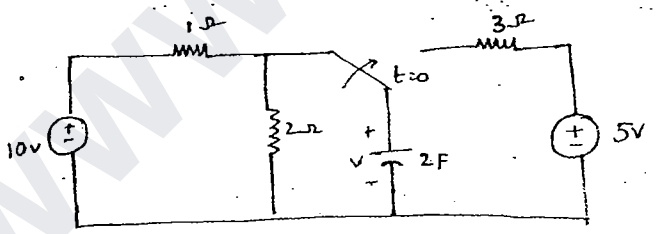
$$V = L \cdot \frac{d}{dt}\left[\frac{V_S}{R}\left(1 - e^{-t/\tau}\right)\right]$$

$$V = \frac{V_S L}{R}\left(0 - e^{-t/\tau} \times \frac{-1}{\tau}\right)$$

$$V_L(t) = \frac{V_S L}{R} \times \frac{1}{\frac{L}{R}}\left(e^{-t/\tau}\right) = V_S e^{-t/\tau}$$



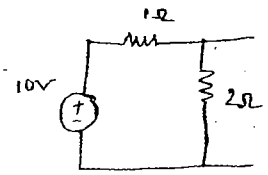
(P)



find $v(t)$ for $t > 0$

$$v(t) = v(\infty) + \left[v(0) - v(\infty)\right] \cdot e^{-t/\tau}$$

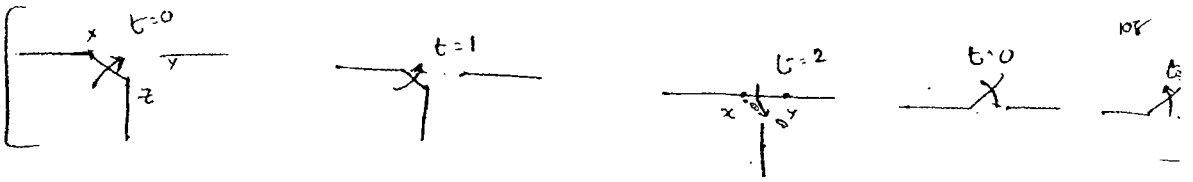
$v(0)$

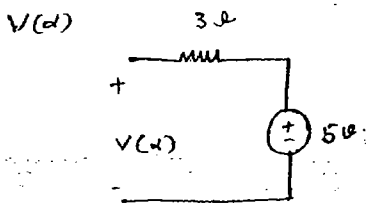


$$v(0) = 10\left[\frac{2}{3}\right]$$

$$= \frac{20}{3}$$

WWW.GATENOTES.IN





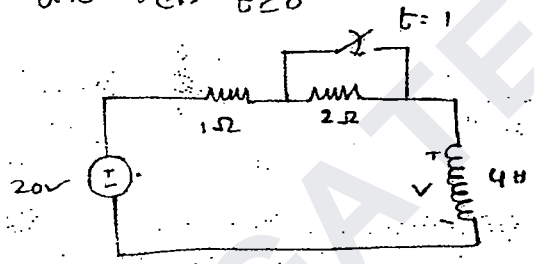
$V(\infty) = 5V$

$\tau = RC = 3 \times 2 = 6$

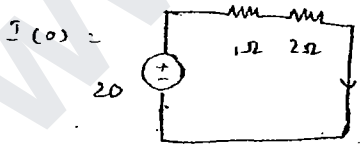
$v(t) = 5 + \left[\frac{20}{3} - 5\right] \cdot e^{-t/6}$

$= 5 + \frac{5}{3} e^{-t/6}$

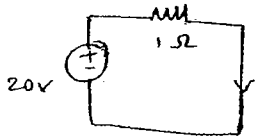
(b) Find $V(t)$ $t \geq 0$



$I(t) = I(\infty) + [I(0) - I(\infty)] e^{-t/\tau}$



$I(0) = \frac{20}{3} A$



$i(\infty) = \frac{20}{1} = 20A$

$\tau = \frac{L}{R} = \frac{4}{1} = 4 sec$

$i(t) = 20 + \left[\frac{20}{3} - 20\right] \cdot e^{-0.25t}$

$= 20 - 13.33 e^{-0.25t}$

WWW.GATENOTES.IN

$t \geq 0$

$\rightarrow \quad 0 < t < 1$

$i(t) = 20/3\ A$

$V = L\frac{di}{dt} = L\frac{d}{dt}(20/3) = 0$

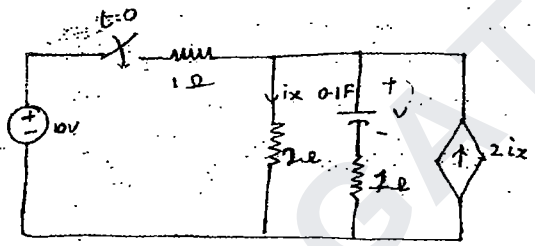
$\rightarrow \quad t \geq 1$

$i(t) = 20 - 13.3\, e^{-0.25(t-1)}$

$V(t) = L\frac{di(t)}{dt} = 4\frac{di(t)}{dt} = -4 \times 13.33\, e^{-(0.25)(t-1)}(-0.25)$

$$V(t) = \begin{cases} 0, & t < 1 \\ 13.33\, e^{-0.25(t-1)}, & t \geq 1 \end{cases}$$

(P)

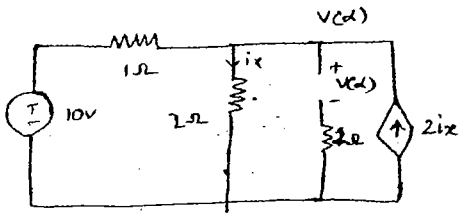


find V $t \geq 0$

$V(t) = V(\infty)\,[V(0) - V(\infty)]\cdot e^{-t/\tau}$

$V(0) = 0$ volts No Source.

$V(\infty)$ Switch operate



$$\frac{V(\infty) - 10}{1} + \frac{V(\infty)}{2} - 2\,i_x = 0 \quad \text{—(1)}$$

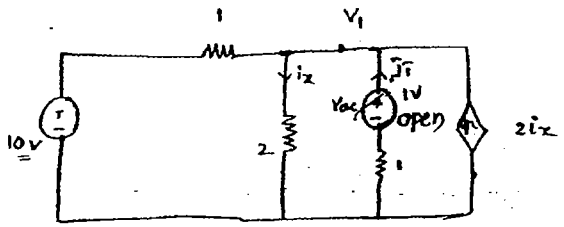
$$i_x = V(\infty)/2 \quad \text{—(2)}$$

WWW.GATENOTES.IN

$$\frac{V(\infty)-10}{1}+\frac{V(\infty)}{2}-\frac{V(\infty)}{\infty}=0$$

$$V(\infty) = 20V$$

$\tau = RC$



$$\frac{V_1-10}{1}+\frac{V_1}{2}+\frac{V_1-0}{\infty}-2i_x=0 \quad \text{—①}$$

$$i_x = V_1/2$$

$$V_1-10+\frac{V_1}{2}+V_1-V_1=0$$

$$\frac{3V_1}{2}=10 \Rightarrow V_1=20/3$$

$$\frac{V_1}{2}=10 \Rightarrow V_1=20V=V_{oc}$$

$$i_T=\frac{(0-V_1(-1))}{1}=\frac{-20/3+1}{1}=-19/3$$

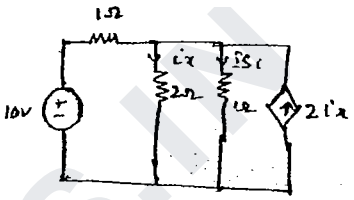
$$i_T=-19/3\ A$$

$$R_{eq}=\frac{V}{i_T}=\frac{1}{-19/3}=-3/19\ \Omega$$

$$\tau = RC = \frac{-3}{19}\times\frac{1}{10}=-3/190\ sec$$

$$V(t)=20+[0-20]e^{+190t/3}\ volts.$$

$$=20\left[1-e^{+63.3t}\right]\ volts.$$



$$\frac{V_1-10}{1}+\frac{V_1}{2}+\frac{V_1}{1}-2i_x=0 \quad \text{—①}$$

$$i_x = V_1/2$$

$$I_{sc}=\frac{V_1}{1}$$

$$R_{eq}=\frac{V_{oc}}{I_{sc}}$$

$$V_1-10+\frac{V_1}{2}+\frac{V_1}{1}-2\frac{V_1}{2}=0$$

$$\frac{3V_1}{2}=10$$

$$V_1=20/3$$

$$I_{sc}=20/3\ A$$

$$R_{eq}=\frac{20}{20/3}=3\ \Omega$$

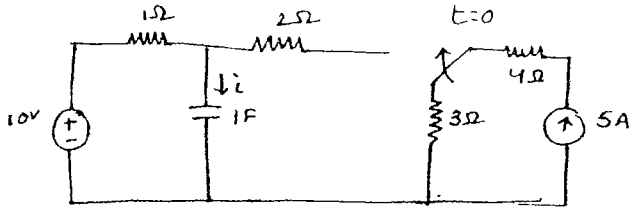
$$\tau = RC = 3\times\frac{1}{10}=0.3\ sec$$

$$V(t)=20+[0-20]e^{-t/0.3}$$

$$=20\left[1-e^{-3.3t}\right]\ volts$$

WWW.GATENOTES.IN

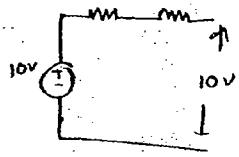
(P)



V(t) is what.

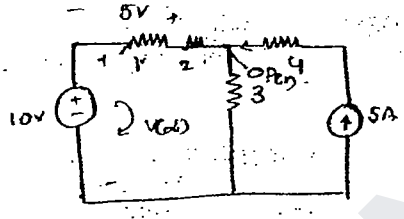
$$i(t) = C\frac{dv(t)}{dt}$$

$V(0^-)$ =



$= \frac{10V}{2}$

$V(\infty)$ =



$\tau = Req \times C = 1 \times 1 = 1$ sec

$-10 - 5 + V(\infty) = 0$

$V(\infty) = 15$

$V(t) = 15 + (10-15)e^{-t}$

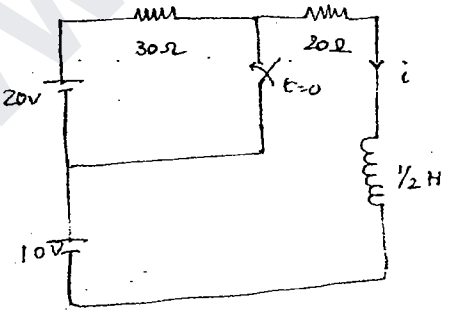
$= 15 - 5e^{-t}$

$i = C\frac{dv}{dt}$

$= 1 \times 5e^{-t}$ Amps

(P)

$i(t) \quad t \geq 0$



$i(0) = \frac{30V}{50\Omega} = 3/5$ A

$i(\infty) = \frac{10V}{20\Omega} = \frac{1}{2}$ A

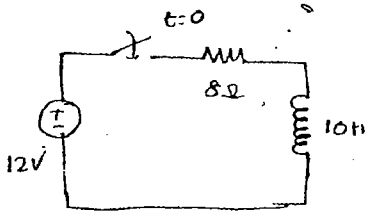
$\tau = \frac{1/2}{20} = \frac{1}{40}$ sec

$i(t) = \frac{1}{2} + \left[\frac{3}{2} - \frac{1}{2}\right]e^{-40t}$

$i(t) = 0.5 + 0.1e^{-40t}$ Amps

WWW.GATENOTES.IN

(P) For the ckt shown below at what time the current is 1.2A



$i(t) = 1.2A$.

$i(0) = 0$

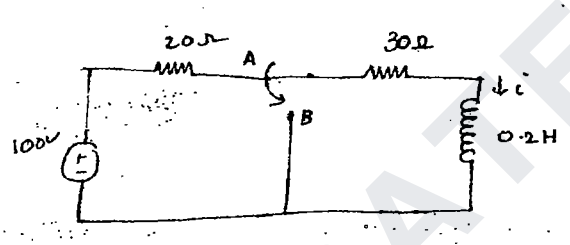
$i(\infty) = \frac{12}{8} = \frac{3}{2}A$.

$i(t) = i(\infty) + (i(0) - i(\infty)) \cdot e^{-t/\tau}$. $\quad \tau = \frac{L}{R}$ $\quad R = \frac{12}{3/2} = \frac{24}{3} = 8$

$1.2 = \frac{3}{2} + (0 - 3/2) \cdot e^{-0.8t}$ $\quad \tau = \frac{10}{8} = 1.25$

$t = 2.01$

(P)



$i(t)$ at $t = 6.66$ msec

$i(0) = \frac{100V}{50\Omega} = 2A$ $\quad i(\infty) = 0$

$\tau = \frac{L}{R}$ $\quad R = 30\Omega$

$i(t) = i(\infty) + [i(0) - i(\infty)] \cdot e^{-t/\tau}$

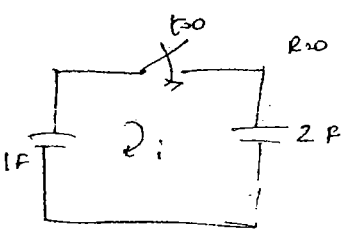
$\tau = \frac{0.2}{30}$

$= 0 + [2 - 0]\, e^{-t/0.2/30}$

$= 2\, e^{-\frac{6.66 \times 10^{-3} \times 30}{0.2}}$

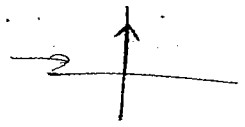
$i(t) = 0.73A$

(P)



a) Step

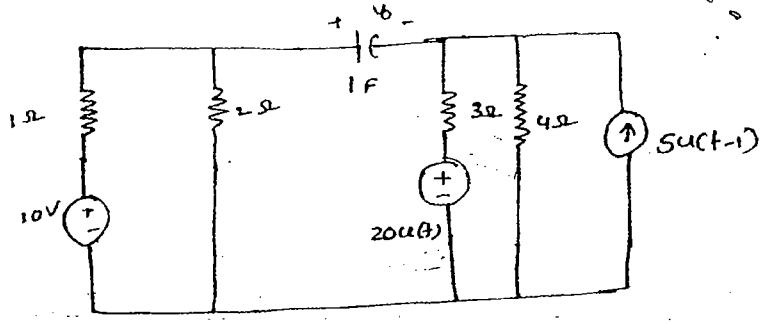
b) impulse

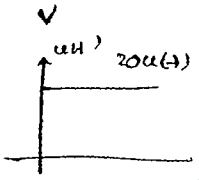


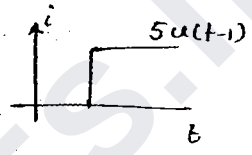
WWW.GATENOTES.IN

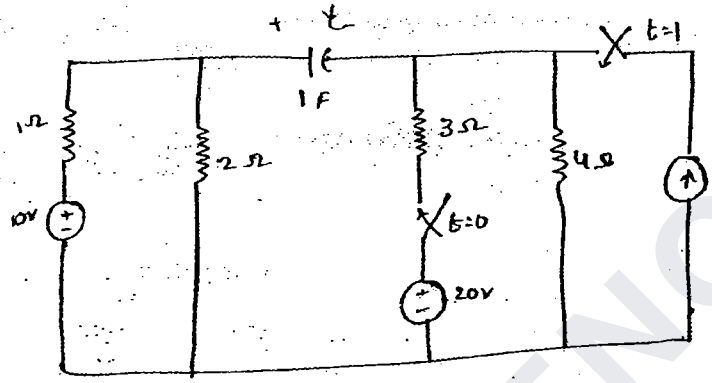
(P)

Find $V_C(t)$ $t \geq 0$



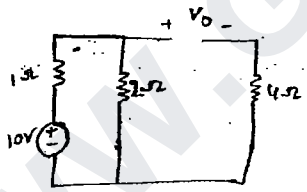




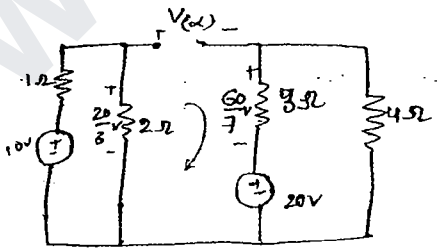


$0 < t < 1$

$$V(t) = V(\infty) + [V(0) - V(\infty)] \cdot e^{-t/\tau}$$



$$V(0) = 10\left[\frac{2}{3}\right] = \frac{20}{3}V$$

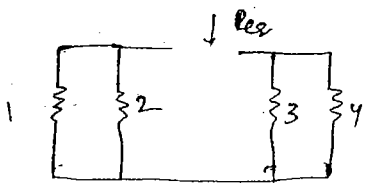


$$-\frac{20}{3} + V(\infty) - \frac{60}{7} + 20 = 0$$

$\left(\frac{20}{3}, \frac{3}{7}\right)$

$$V(\infty) = \frac{20}{3} + \frac{60}{7} - 20$$

$$= -\frac{100}{21}$$



$$R_{eq} = [1 \| 2] + [3 \| 4]$$

$$= \frac{2}{3} + \frac{12}{7} = \frac{50}{21}\Omega$$

WWW.GATENOTES.IN

$\tau = RC$

$= 1 \times \frac{50}{21} = \frac{50}{21}$ sec

$$v(t) = -\frac{100}{21} + \left[20/3 + \frac{100}{21} \right] e^{-t/\frac{50}{21}}$$

$t \geq 1$

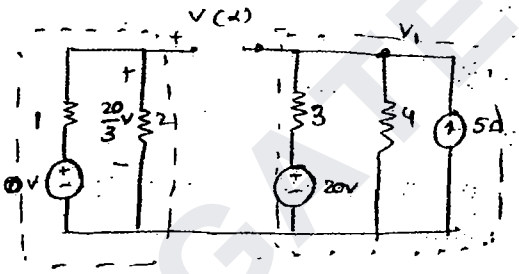
$$v(t) = v(\infty) + [v(0) - v(\infty)] \cdot e^{-t/\tau}$$

$v(0)$: ⟶ is $v(1)$ from previous case.

$$v(0) = v(1) = -\frac{100}{21}\left[\frac{20}{3} + \frac{100}{21}\right] \cdot e^{-21/50}$$

$$= +2.74 \text{ V}$$

$v(\infty)$:



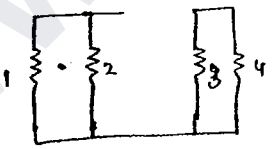
$$\frac{V_1 - 20}{3} + \frac{V_1}{4} - 5 = 0$$

$$V_1 = 20\text{V}$$

$$-\frac{20}{3} + v(\infty) + 20 = 0$$

$$v(\infty) = -\frac{40}{3} \text{ V}$$

τ



$$\tau = RC = [(1 \| 2) + (3 \| 4)] \times 1\text{F}$$

$$= 50/21 \text{ sec}$$

$$\tau = \frac{50}{21} \text{ sec}$$

$$v(t) = -\frac{40}{3} + \left[2.74 + \frac{40}{3} \right] \cdot e^{-\frac{21(t-1)}{50}} \quad \text{for } t \geq 1$$

$$= -\frac{100}{21} + \left[\frac{20}{3} + \frac{100}{21} \right] e^{-t/\frac{50}{21}} \quad \text{for } 0 < t < 1$$

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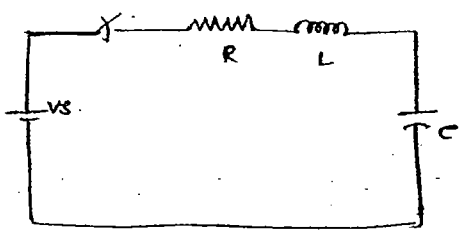
Step Response of 2nd order :-

Series R, L, C



Two State Variables 2nd order

Best State Variable 'V' → voltage across Capacitor

Case (a) $\alpha > \omega_0$ — over damped Response

$$V(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}$$

$\alpha = \frac{R}{2L}$ $\omega_0 = \frac{1}{\sqrt{LC}}$

Case (b) $\alpha = \omega_0$ — critically damped

$$V(t) = (A_1 + A_2 t) e^{-\alpha t}$$

Case c: $\alpha < \omega_0$ — underdamped

$$V(t) = e^{-\alpha t} [A_1 \cos \omega_d t + A_2 \sin \omega_d t]$$

$$\omega_d = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

Total Response of 2nd order ckts

$$V(t) = V_{tr}(t) + V_{s.s}(t)$$

$V_{tr}(t)$ → Transient

$V_{s.s}(t)$ → Steady state

$$V(t) = V_s + A_1 e^{S_1 t} + A_2 e^{S_2 t} \quad \text{(over damped)}$$

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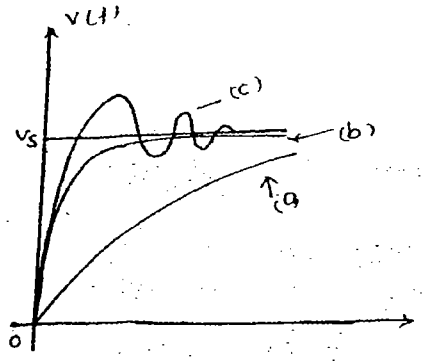
(b) $V(t) = V_S + (A_1 + A_2)e^{-\alpha t}$ (Critically damped)

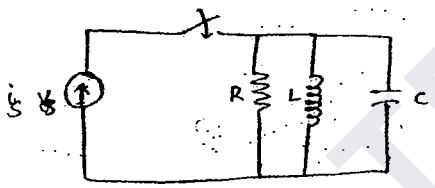
(c) $V(t) = V_s + e^{-\alpha t}[A_1 \cos\omega_d t + A_2 \sin\omega_d t]$ (under damped)



$\alpha = \frac{R}{2L}$ $\omega_0 = \sqrt{\frac{1}{LC}}$

$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

(b) Step Response of parallel RLC ckt



→ Two State Variable ⇒ 2nd Order System

Best State Variable is i - Current through Inductor

$i(\infty) = i_s$

Case (a) $\alpha > \omega_0$ → over damped Response

$$i(t) = A_1 e^{+s_1 t} + A_2 e^{s_2 t}$$

Case (b)

$\alpha = \omega_0$ → Critically damped

$$i(t) = (A_1 + A_2 t)e^{-\alpha t}$$

Case (c)

$\alpha < \omega_0$ → under damped

$$i(t) = e^{-\alpha_1 t}[A_1 \cos\omega_d t + A_2 \sin\omega_d t]$$

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Ravi's R Single
Ekekrabarthi

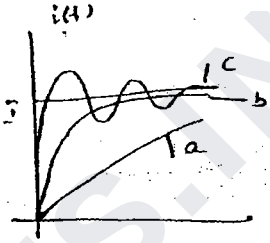
Total Response of 2nd Order ckt

$$i(t) = i_{ss}(t) + i_{tr}(t)$$

$$i(t) = \bar{y}_s + A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad \text{(over damped)}$$

$$i(t) = \bar{y}_s + (A_1 + A_2 t) e^{-\alpha t} \quad \text{(critically damped)}$$

$$i(t) = \bar{y}_s + (A_1 \cos \omega_d t + A_2 \sin \omega_d t) e^{-\alpha t} \quad \text{(under damp)}$$



$$\alpha = \frac{1}{2RC} \qquad \omega_0 = \frac{1}{\sqrt{LC}} \qquad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

Initial Condition problems:-

Ex: $\frac{di}{dt}(0^+)$ $\frac{d^2v}{dt^2}(0^+)$ $i(0^-)$, $i(0^+)$

$t = 0^-$ → instant just before switch operation

$t = 0^+$ → just after S/w operation

$t = 0$ → Exact instant of s/w operation

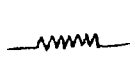
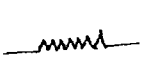
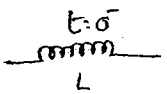
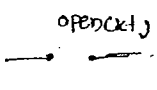
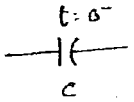
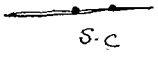
- A capacitor never allows sudden change in voltage

$$v_c(0^-) = v_c(0) = v_c(0^+)$$

- An inductor never allows sudden change in current

$$i_L(0^-) = i_L(0) = i_L(0^+)$$

Equivalent ckt Representation of passive elements at $t = 0^+$ (transient state)

	$t = 0^-$	$t = 0^+$	
R			(memory less unit)
L			$Z_L = sL$
C			

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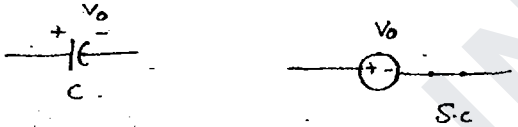
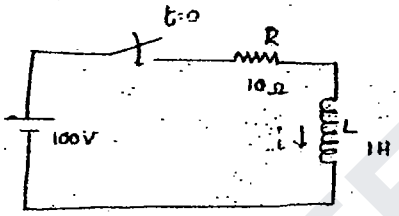
L with initial Current I_0 $\quad t=0^-$ $\quad t=0^+$

C with initial voltage V_0



Capacitor never allow Sudden change of voltage

(P) For the ckt shown find $i(0^+)$, $\frac{di}{dt}(0^+)$, $\frac{d^2}{dt}(i(0^+))$ ← steady state ↑ velocity ↑



$t=0^-$ S/w open 'i' can't flow so $i(0^-)=0$.

Inductor Never allows Sudden change in Current

$$i(0^+) = i(0^-)$$

$$\boxed{i(0^+) = 0 \text{ A}}$$

KVL

$$-100 + 10i + 1\cdot\frac{di}{dt} = 0 \quad \text{—— ①}$$

at

$$-100 + 10\,i(0^+) + \frac{d}{dt}i(0^+) = 0$$

($i(0^+) \uparrow 0$)

$$\boxed{\frac{di(0^+)}{dt} = 100 \text{ A/Sec}}$$

differentiating ①

$$0 + 10\frac{di}{dt} + \frac{d^2i}{dt^2} = 0$$

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$$10\frac{d}{dt}i(0^+) + \frac{d^2}{dt}(i(0^+)) = 0$$

$$\frac{d^2}{dt}i(0^+) = -10\frac{di(0^+)}{dt}$$

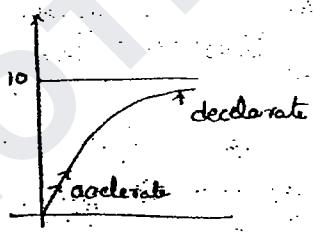
$$\frac{d^2}{dt}i(0^+) = -10\times 100 = -1000\ A/Sec^2$$

For 1st order Step Response

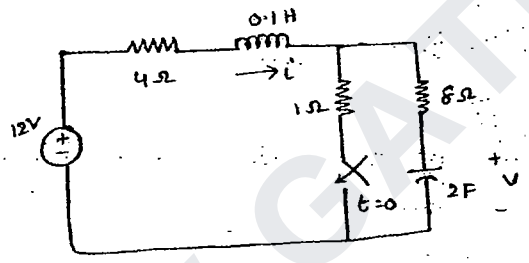
$$i(t) = I(\infty) + [I(\infty) - I(0)]e^{-t/\tau}$$

$$= 10 + [0 - 10]e^{-10t}$$

$$= 10\left[1 - e^{-10t}\right]$$

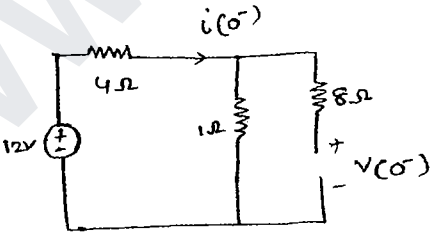


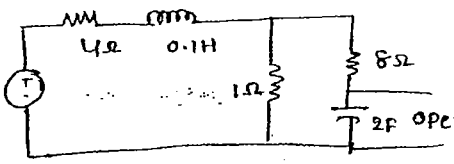
(2)



For the ckt Find $i(0^+)$, $V(0^+)$, $\frac{dV(0^+)}{dt}$, $\frac{di(0^+)}{dt}$, $i(\infty)$, $V(\infty)$

Ckt before at $t = 0^-$





$$i(0^-) = \frac{12}{5}\ A \longrightarrow i(0^+) = 12/5\ A$$

$$V(0^-) = 12\left(\frac{1}{5}\right) = \frac{12}{5}\ V \qquad V(0^+) = \frac{12}{5}\ V$$

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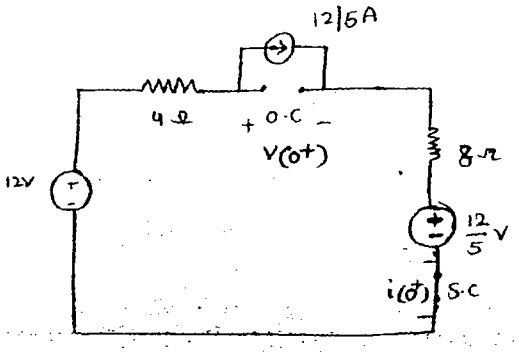
ckt at $t = 0^+$



$i_c(0^+) = \frac{12}{5}$ A

KVL

$$-12 + 4\left[\frac{12}{5}\right] + V(0^+) + 8\left[\frac{12}{5}\right] + \frac{12}{5} = 0$$

$$V_L(0^+) = -\frac{96}{5} V$$

So

$$\frac{di(0^+)}{dt} = \frac{V(0^+)}{L}$$

$$= \frac{-96}{5} \times 0.1 = -\frac{960}{5} = -192 \text{ A/sec}$$

$$\boxed{\frac{di(0^+)}{dt} = -192 \text{ A/sec}}$$

$$\rightarrow \frac{dV(0^+)}{dt} = \frac{i_c(0^+)}{C} = \frac{12}{5 \times 2} = 1.2 \text{ V/sec}$$

$$\boxed{\frac{dV(0^+)}{dt} = 1.2 \text{ V/sec}}$$

Ckt at $t \to \infty$

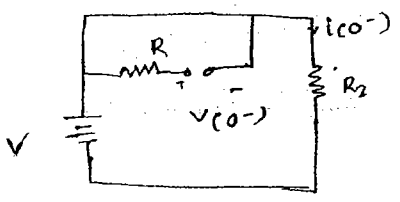
$i(\infty) = 0$

$V(\infty) = 12V$

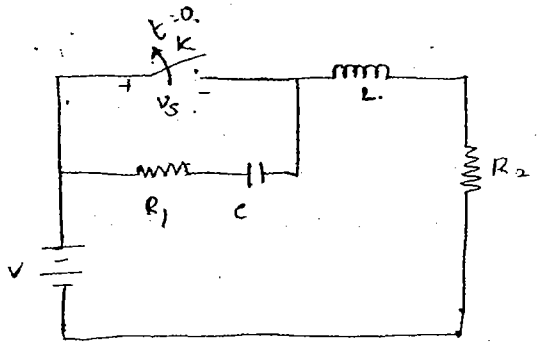
2M Gate 2007

If Switch 'K' opened at $t = 0$ then find $V_S(0^+)$

Ckt at $t = 0^-$

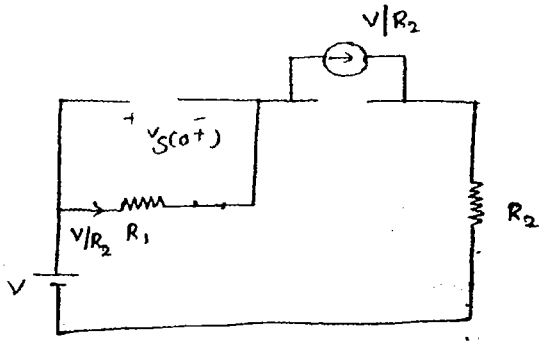


$V(0^-) = 0$ $\quad i(0^-) = V/R_2$



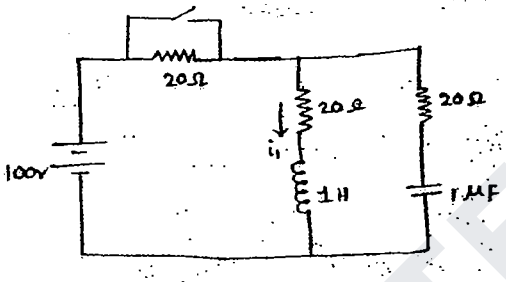
WWW.GATENOTES.IN

ckt at $t = 0^+$



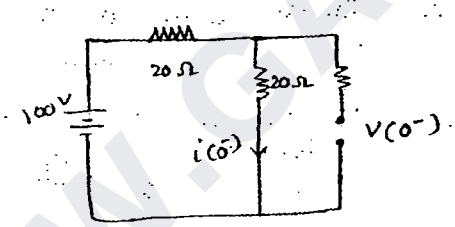
$$V_S(0^+) = \frac{V}{R_2} \cdot R_1 = \frac{VR_1}{R_2}$$

2004
ECE



The Switch is closed at $t=0$ find $\frac{d}{dt}(i_1(0^+))$

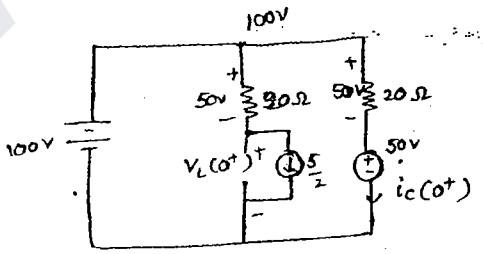
at $t = 0^-$



$$i(0^-) = \frac{100}{40} = 5/2\ A$$

$$V(0^-) = 50V$$

at $t = 0^+$



$$-V_L(0^+) - 50V + 50 + 50 = 0$$

$$V_L(0^+) = 50V$$

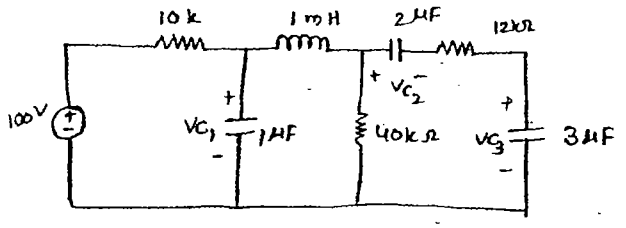
$$V_L(0^+) = L\frac{di(0^+)}{dt}$$

$$\frac{di(0^+)}{dt} = \frac{V_L(0^+)}{L} = \frac{50}{1}\ A/sec$$

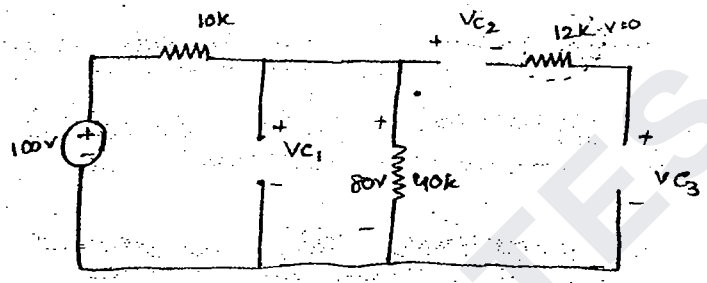
ckt at $t = 0^+$

WWW.GATENOTES.IN

* determine the steady state voltages across the Capacitor



at $t = 0^-$

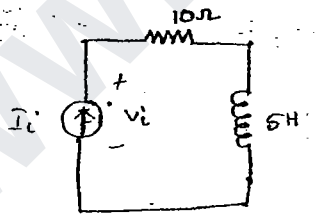


$$V_{C_1} = 100\left[\frac{40k\Omega}{40k\Omega + 10k\Omega}\right] = 100 \times \frac{40}{50} = \underline{\underline{80V}}$$

$$V_{C_2} = 80V\left[\frac{3\mu F}{5\mu F}\right] = 48V$$

$$V_{C_3} = 80V\left[\frac{2\mu F}{5\mu F}\right] = \underline{\underline{32V}}$$

* If $I_i = 0.4t^2$ then $V_i(t)$ at $t = 1sec$ is



KVL

$$-V_i + 10I_i + 5\frac{dI_i}{dt} = 0$$

$$V_i(t) = 10I_i(t) + 5\frac{dI_i}{t}$$

$$= 10(0.4)t^2 + 5\frac{d}{dt}(0.4t^2)$$

$$= 4t^2 + 5\,0.8t$$

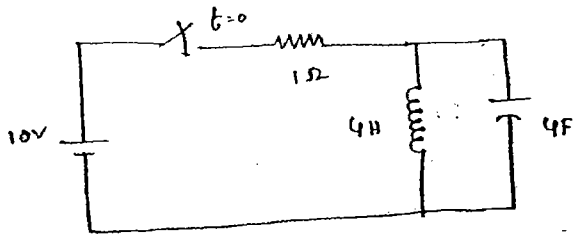
$$= 4(t + t^2)$$

at $t = 1$

$$V_i(1) = 4(1+1) = 8 \text{ volts}$$

WWW.GATENOTES.IN

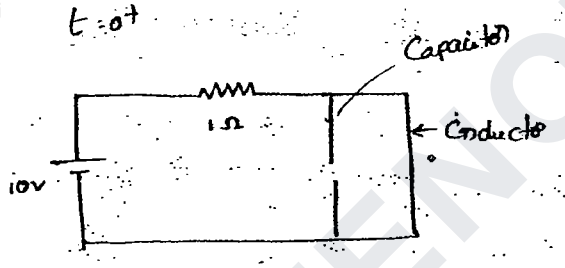
* determine Current through the Battery at $t=0^+$ & $t \to \infty$



at ckt $t=0^-$

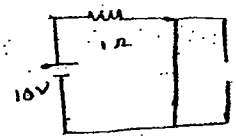
$i(0^-) = 0\ A \quad v(0^-) = 0$

at ckt $t=0^+$



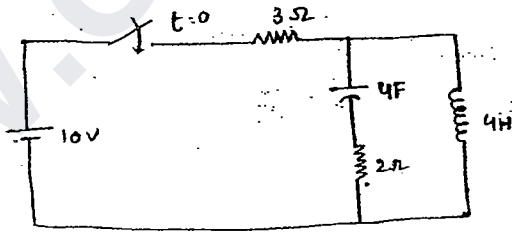
$i(0^+) = 10A$

at $t=\infty$

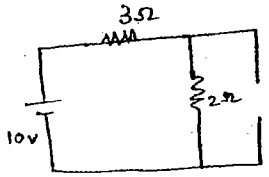


$i(\infty) = 10A$.

* determine Current through Battery $t=0^+$ & $t=\infty$

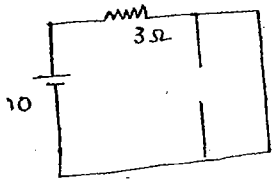


Ck $t=0^+$



$i(0^+) = \frac{10}{5} = 2A$

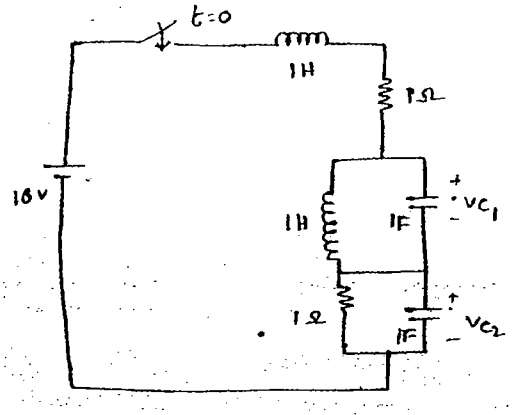
$t=\infty$



$i(\infty) = 10/3\ A$

WWW.GATENOTES.IN

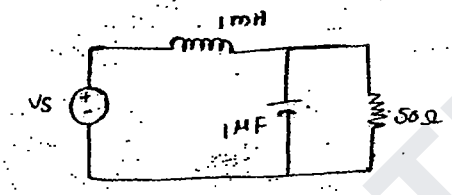
Gate 2009 determine Steady State Capacitor voltages

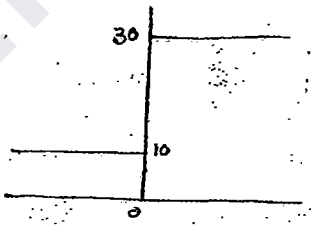


V_{C_1} = 0 volt

$V_{C_2} = 10\left(\frac{1}{2}\right) = 5V$

Gate 2009 If, $V_S = 10 + 20\,u(t)$ Find $\frac{di_L(0^+)}{dt}$, $\frac{dv_C(0^+)}{dt}$



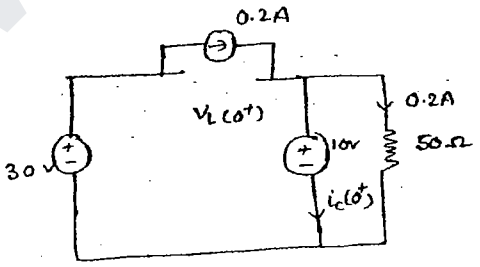


$t = 0^-$

$i(0^-) = \frac{10}{50} = 1/5 = 0.2A$

$V(0^-) = 0.2 \times 50 = 10V$

Ckt $t = 0^+$



KVL

$-30 + V_L(0^+) + 10 = 0$

$V_L(0^+) = 20V$

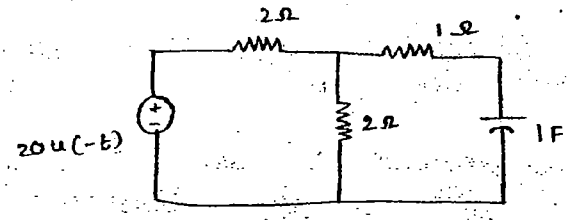
$i_c(0^+) = 0\ A$

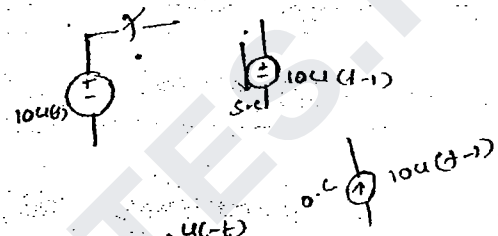
WWW.GATENOTES.IN

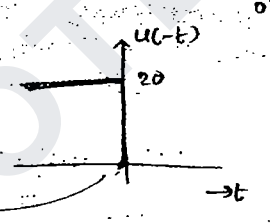
$$\frac{di_L(0^+)}{dt} = \frac{V_L(0^+)}{L} = \frac{20}{1mH} = 20kA/sec$$

$$\frac{dV_C(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{0}{C} = 0V/sec$$

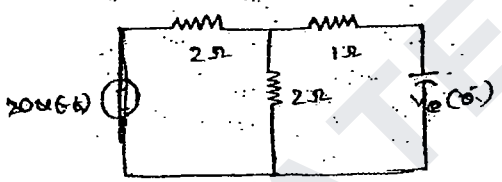
(P) Find $v_c(t)$ & $i_c(t)$ $\}$ $t \geq 0$







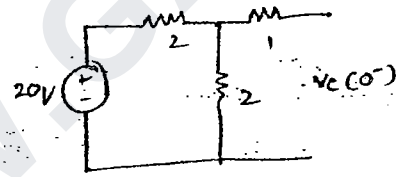
at $t = 0^+$ Voltage Source Shorts



Source free RC ckt

$$v(t) = v_0 \cdot e^{-t/\tau}$$

at $t = 0^-$



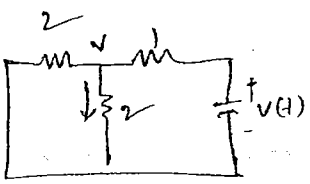
$v_c(0^-) = 10$ volts

$v_c(0^+) = v_c(0^-)$

$v_c(0^+) = 10$ volts

at $t = 0^+$ $\quad v(t) = v_0 e^{-t/\tau}$

$$= 10\, e^{-0.5t} \text{ volt}$$



$$i(t) = C\frac{dv}{dt}$$

$$= 1 \frac{d}{dt}\left[10\, e^{-0.5t}\right]$$

$$i_c(t) = -5 \cdot e^{-0.5t} \text{ A}$$

$$\frac{v_1}{2} + \frac{v_1}{2} + \frac{v_1 - v(t)}{1} = 0$$

$$v_1 = 5\, e^{-0.5t}, \quad \frac{v_1}{2} =$$

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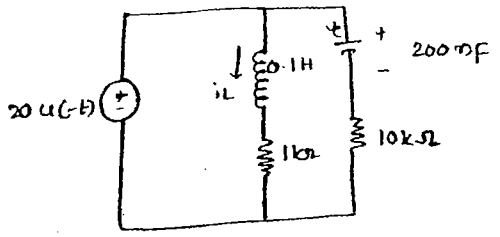
* IES determine v_c & i_L $t \geq 0$



Find complete exp for i_L & v

$\frac{\tau_v}{\tau_i}$

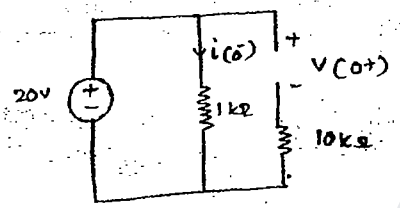
$\tau_i = \frac{0.1}{1\times10^3}$ $\quad \tau_v = 10\times10^3\times200\times10^{-9}$

$\left\{ \frac{\tau_v}{\tau_i} = \frac{2\times10^{-3}}{0.1}\times10^3 \right.$

$= 20$

② at $t = 0^-$

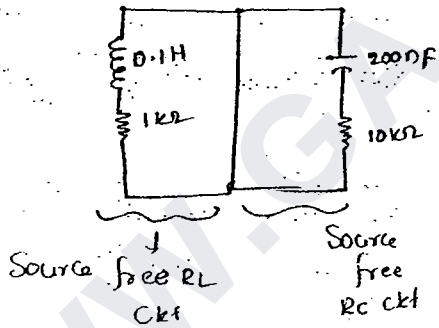
u(t) is short $t = 0^+$



$i(0^-) = 20\text{ mA} \quad v(0^-) = 20\text{v} = v(0^+) = v_0$

$= i(0^+) = i_0$

$t \to 0^+$ Source free.



$i(t) = I_0 \cdot e^{-t/\tau}$

$\tau = \frac{L}{R} = \frac{0.1}{1k} = 100\mu\text{sec}$

$\Rightarrow 20\times10^{-3}\, e^{-t/100\mu}$

$v(t) = v_0\, e^{-t/\tau}$

$\tau = RC = 10k \times 200\times nF = 2\text{ msec}$

$v(t) = 20\, e^{-500t}$ volts

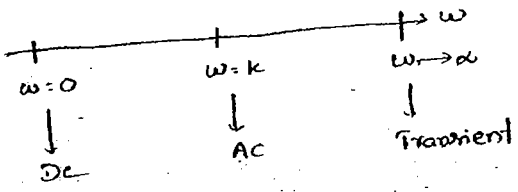
$\boxed{v(t) = 20\, e^{-500t}}$

$\frac{\tau_L}{\tau_C} = \frac{20\times10^{-3}}{2m}\, e^{\frac{100\mu}{}}$

$= 50\text{ msec}$

WWW.GATENOTES.IN

Laplace Method



frequency domain

$$L\{f(t)\} = F(s) = \int_0^\infty e^{-st} f(t)\,dt$$

$V = \frac{v}{s}$

$I = \frac{i}{s}$

$v(t) = V(s)$

$i(t) = I(s)$

$V_m \sin\omega t \rightarrow \frac{V_m \cdot \omega}{s^2+\omega^2}$

$V_m \cos\omega t \rightarrow \frac{V_m s}{s^2+\omega^2}$

$R \longrightarrow R$

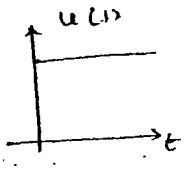
$L \longrightarrow +j\omega L \longrightarrow sL$

$C \longrightarrow \frac{1}{j\omega C} \longrightarrow \frac{1}{sC}$

Singularity functions:-

unit Step:-

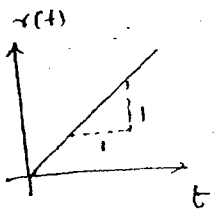
$$u(t) \triangleq \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$$



$$L\{u(t)\} = 1/s$$

unit Ramp i/p:-

$$r(t) \triangleq \begin{cases} t, & t>0 \\ 0 & t<0 \end{cases}$$



$$L\{r(t)\} = 1/s^2$$

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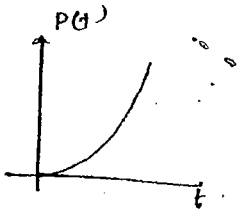
unit parabolic :-

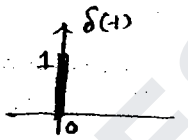
$$P(t) \triangleq \begin{cases} \frac{t^2}{2} & t > 0 \\ 0 & t < 0 \end{cases}$$



$$\mathcal{L}\{P(t)\} = 1/s^3$$

unit impulse :-

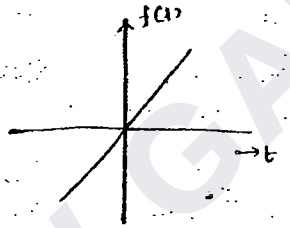
$$\delta(t) \triangleq \begin{cases} 1, & t = 0 \\ 0, & t \neq 0 \end{cases}$$



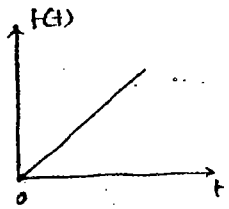
$$\mathcal{L}\{\delta(t)\} = 1$$

Any function multiplied by $u(t)$ means ie function is defined for $t > 0$.

Ex: $f(t) = t$

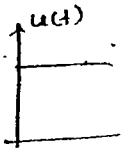


Ex: $f(t) = t\,u(t)$

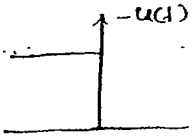


plot

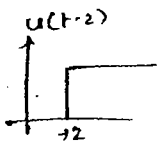
a) $u(t)$



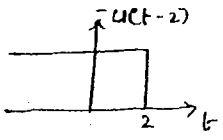
$+u(t)$



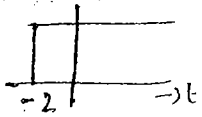
b) $u(t-2)$



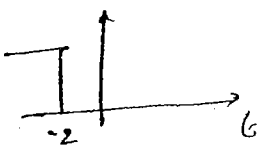
$+u(t+2)$



c) $u(t+2)$



$+u(-t+2)$



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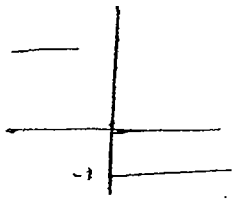
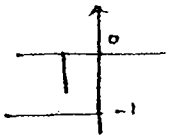
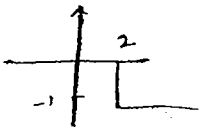
- $u(+t)$



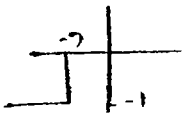
- $u(-t)$



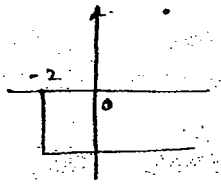
- $u(+t-2)$



- $u(-t-2)$



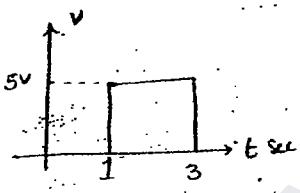
- $u(+t+2)$



- $u(-t+2)$

(P) Find L.T. of Signal...

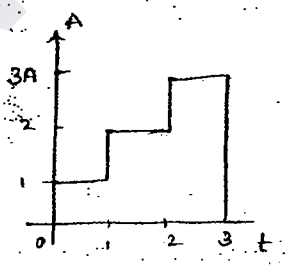
(1)



$V(t) = 5\,u(t-1) - 5\,u(t-3)$

$= \frac{5}{s} e^{-s} - \frac{5}{s} e^{-3s}$

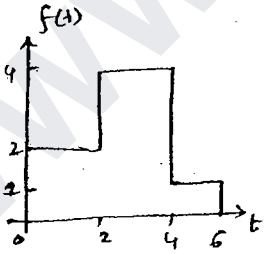
(P)



$i(t) = u(t) + u(t-1) + u(t-2) - 3u(t-3)$

$= \frac{1}{s} + \frac{e^{-s}}{s} + \frac{e^{-2s}}{s} - \frac{3e^{-3s}}{s}$

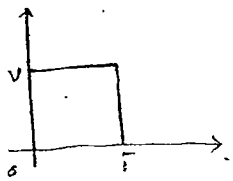
(P)



$f(t) = 2u(t) + 2u(t-2) - 4u(t-4) - u(t-6)$

$= \frac{2}{s} + \frac{2}{s}e^{-2s} - \frac{3}{s}e^{-4s} - \frac{1}{s}e^{-6s}$

(P)



$f(t) = V u(t) - V u(t-T)$

$f(s) = V\left[\frac{1}{s} - \frac{e^{-Ts}}{s}\right]$

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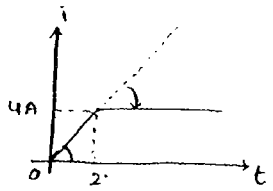
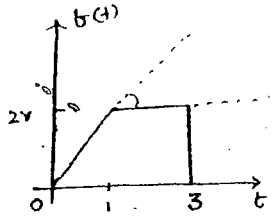
(P)



$i(t) = 2t\,u(t) - 2(t-2)\,u(t-2)$

$= \frac{2}{s^2} - \frac{2}{s^2}e^{-2s}$

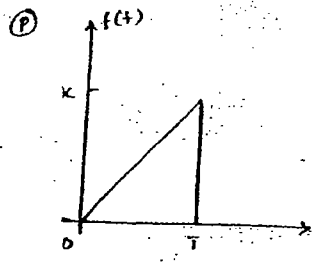
(P)



$b(t) = 2t\,u(t) - 2(t-1)u(t-1) - 2\,u(t-3)$

$= \frac{2}{s^2} - \frac{2e^{-s}}{s^2} - \frac{2}{s}e^{-3s}$

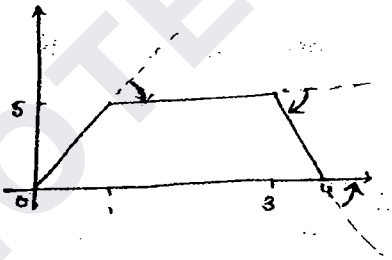
(P)



$f(t) = \frac{K}{T}t\,u(t) - \frac{K}{T}u(t-T)(t-T) - K\,u(t-T)$

$F(s) = \frac{K}{T}\left[\frac{1}{s^2} - \frac{e^{-Ts}}{s^2} - Te^{-Ts}\right]$

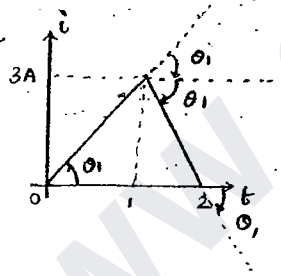
(P)



$v(t) = 5t\,u(t) - 5(t-1)u(t-1) - 5(t-3)\,u(t-3) + 5(t-4)u(t-$

$v(t) = \frac{5}{s^2} - \frac{5}{s^2}e^{-s} - \frac{5}{s^2}e^{-3s} + \frac{5}{s^2}e^{-4s}$

IES



$\Rightarrow 3t\,u(t) - 6(t-1)u(t-1) + 3(t-2)\,u(t-2) = i(t)$

$I(s) = \frac{3}{s^2} - \frac{6}{s^2}e^{-s} + \frac{3}{s^2}e^{-2s}$

Laplace Transforms of periodic function:

$$F(s) = F_1(s)\left[\frac{1}{1-e^{-Ts}}\right]$$

where $F_1(s) \rightarrow$ L.T of 1st Time period

$T \rightarrow$ Time period of Signal

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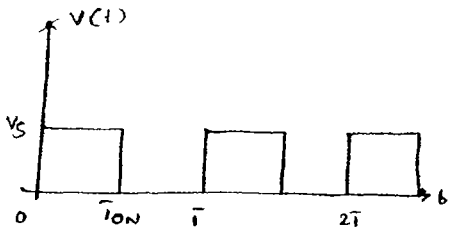
(P)

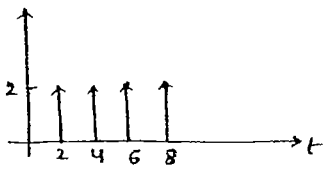


$f_1(t) = V_S u(t) - V_S u(t - T_{ON})$

$F_1(S) = \frac{V_S}{S} - \frac{V_S}{S} e^{-T_{ON}S}$

$F(S) = \frac{\frac{V_S}{S}\left(1 - e^{-T_{ON}S}\right)}{\left(1 - e^{-TS}\right)}$

(P)



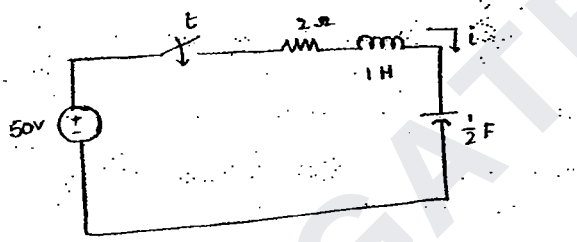
$f_1(t) = 2\delta(t)$

$F_1(S) = 2$

$F(S) = \frac{2}{1 - e^{-2S}}$

IES

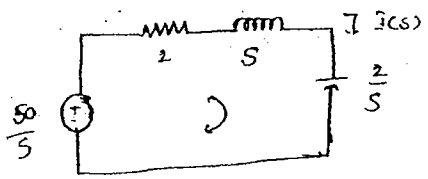
In the RLC ckt shown there is no initial charge of Capacitor Switch A is closed at $t=0$: obtain the Expression for Current i.



Sol → 2nd order Complete solution

use Laplace Transform

Analysing a transient part of at $t=0$ in time domain is analysing the same problem as $S \to \infty$ in frequency domain. ie Steady state analysis in frequency domain



$-\frac{50}{S} + \left(2 + S + \frac{2}{S}\right)\bar{I}(S) = 0$

$\bar{I}(S)\left[\frac{S^2 + 2S + 2}{S}\right] = \frac{50}{S}$

$\bar{I}(S) = \frac{50}{S^2 + 2S + 2}$

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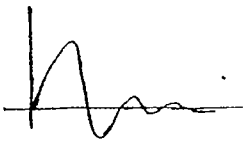
$$I(s) = \frac{50}{(s+1)^2 + (1)^2}$$

$$i(t) = \mathcal{L}^{-1}[I(s)] = 50\, e^{-t} \sin t$$



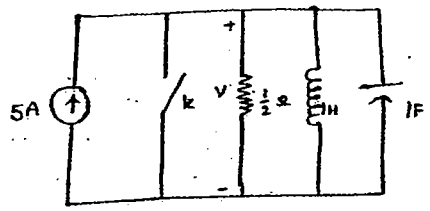
Method 2:

$$\alpha = \frac{R}{2L} = \frac{2}{2} = 1$$

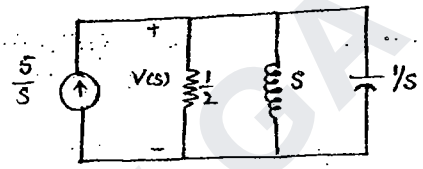
$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 1/2}} = \sqrt{2} \qquad \alpha < \omega_0 \text{ under damped}$$

2006

(P) For the ckt Find the voltage across inductor if Switch is open at $t = 0$



Switch is open



$$\Rightarrow -\frac{5}{s} + 2V(s) + \frac{V(s)}{s} + sV(s) = 0$$

$$V(s)\left[s + \frac{1}{s} + 2\right] = 5/s$$

$$V(s) = \frac{5}{s^2 + 2s + 1}$$

$$V(s) = \frac{5}{(s+1)^2}$$

$$V(t) = 5\, t\, e^{-t}$$

Method 2:

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times \frac{1}{2} \times 1} = 1$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 1}} = 1$$

Critically damped

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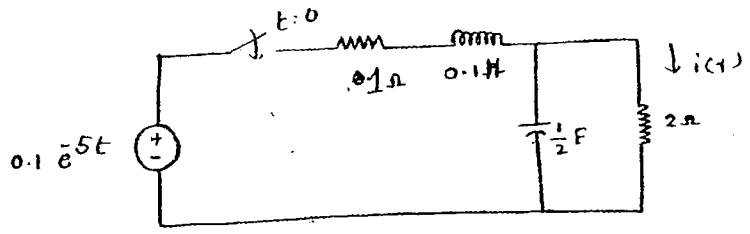
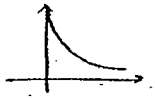
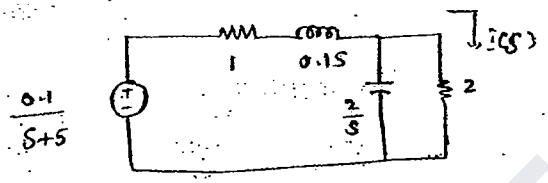
IES
2003 Find the Complete Exp for $i(t)$



→ input is neighter Ac Nor Dc transient 

→ It is 2nd order but it is not Simple Series or parallel RLC.

at $S \to \infty$ Switch is closed.



$$\frac{I}{I}(S) = \frac{V(S)}{Z(S)}$$

$$\frac{I}{I}(S) = \frac{\frac{0.1}{S+5}}{\frac{0.1S^2+1.1S+3}{S+1}}$$

$$= \frac{(0.1)\,S+1}{(S+5)(0.1S^2+1.1S+3)}$$

$$= \frac{S+1}{(S+5)(S^2+11S+30)}$$

$$\frac{I}{I}(S) = \frac{S+1}{(S+5)^2\,(S+6)}$$

$$Z(S) = (1+0.1S) + (2\,\|\,2/S)$$

$$= (1+0.1S) + \frac{4/S}{\frac{2S+2}{S}}$$

$$= (1+0.1S) + \frac{4}{2S+2}$$

$$= \frac{(0.1S+1)(2S+2)+4}{2S+2}$$

$$Z(S) = \frac{0.1S^2+1.1S+3}{S+1}$$

$$I(S) = \frac{I}{I}(S)\left[\frac{2/S}{\frac{2S+2}{S}}\right] = \frac{I}{I}(S)\left[\frac{1}{S+1}\right]$$

$$I(S) = \frac{1}{(S+5)^2(S+6)}$$

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$$I(s) = \frac{A}{s+5} + \frac{B}{(s+5)^2} + \frac{C}{s+6}$$

$C = 1$

$B = 1$

$A = -1$

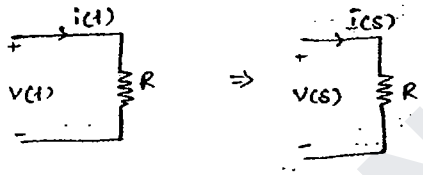
$$I(s) = \frac{-1}{s+5} + \frac{1}{(s+5)^2} + \frac{1}{s+6}$$

$$i(t) = \mathcal{L}^{-1}[I(s)] = -e^{-5t} + te^{-5t} + e^{-6t}$$

$$i(t) = te^{-5t} + e^{-6t} - e^{-5t}$$

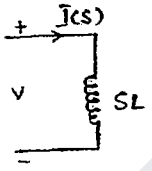
Equivalent ckt Representation of passive elements in Laplace Transform

R →



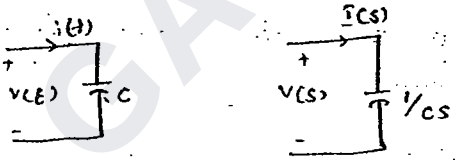
$V(s) = I(s)R$ or $I(s) = \frac{V(s)}{R}$

L →



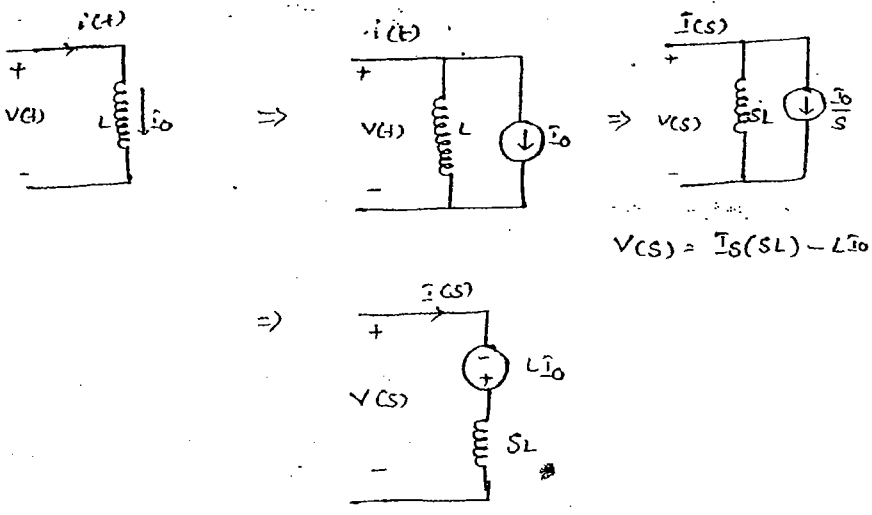
$V(s) = I(s)[SL] \Rightarrow I(s) = \frac{V(s)}{SL}$

C →



$V(s) = \frac{I(s)}{Cs} \Rightarrow I(s) = CsV(s)$

L with initial I_0



$V(s) = I_s(sL) - LI_0$

$V(s) = I(s)[SL] - LI_0$ (or)

$$I(s) = \frac{V(s)}{SL} + \frac{I_0}{s}$$

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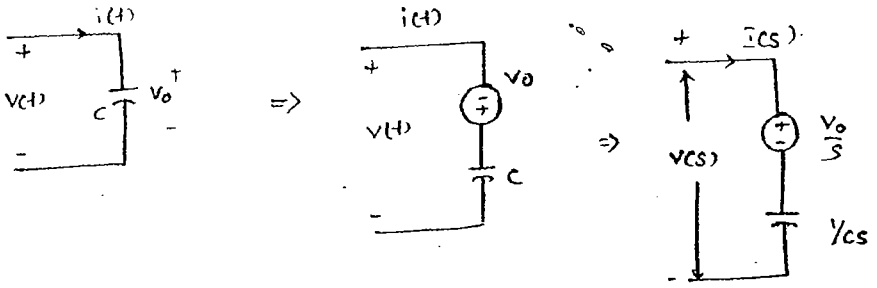
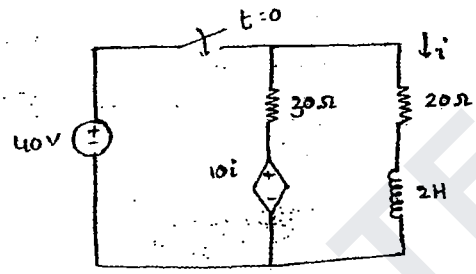
C with V_0 initial V_0



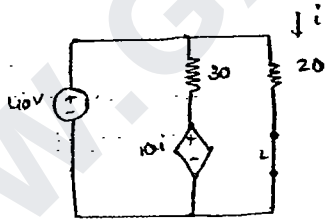
$$V(s) = \frac{I(s)}{cs} + \frac{V_0}{s} \quad \text{i.e.} \quad I(s) = cs\,V(s) - V_0 c$$

(P) For the ckt shown find $i(t)$ for $t \geq 0$

(a) By using time domain (b) Laplace Transform method.



ckt $t = 0^-$



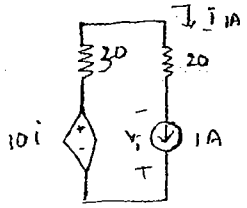
$i(0^-) = 2A$

$I_0 = 2A$

ckt $t = 0^+ \rightarrow$ Source free RL ckt

$$i(t) = I_0 e^{-t/\tau}$$

So $\tau = \frac{L}{R_{eq}}$ R_{eq} by $\frac{V}{I}$ method (Category - III)



KVL $\quad -10 + 50 - \frac{V}{I} = 0$

$V_I = 40\Omega$

$R_{eq} = \frac{V_I}{I} = 40\Omega$

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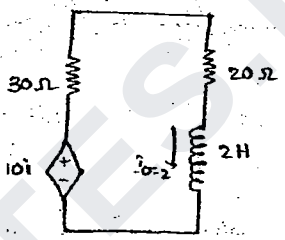
$$\tau = \frac{L}{Req} = \frac{2}{40} = \frac{1}{20}\ sec$$

$$i(t) = 2\ e^{-20t}$$

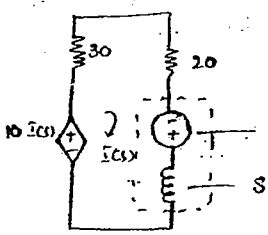
using Laplace Transform

check for initial Current

S/w closed $\quad I(0) = 2A.$



Equivalent ckt in Laplace Transform



KVL $\quad -10\ I(s) + 50\ I(s) - 4 + 2S\ I(s) = 0$

$$I(s)\left[40 + 2S\right] = 4$$

$$I(s) = 4 / 40+2S = \frac{2}{S+20}$$

$$I(t) = 2 \cdot e^{-20t}$$

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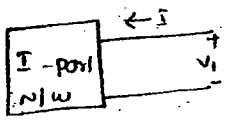
Two port Networks

A port is a pair of terminals through which Voltage Can be Sensed & Current Can be measured

one port Network : It consists of one pair of N/w terminals where posive reference voltage and Current direction are shown

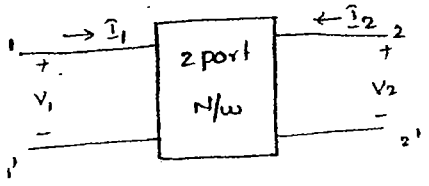


out of two parameters one is Consider Voltage - Current as dependent and other is independent. So we have two parameters to be defined

$$V = ZI \Rightarrow Z = \frac{V}{I}\ \Omega$$

$$I = YV \Rightarrow Y = \frac{I}{V}\ \mho$$

Two port Network :- It consists of two pairs of terminals where +ve reference polarity and Current direction are shown



Here out of 4 parameters I_1, I_2, V_1, V_2 Two Considered as dependent other two Independent So we have total $4C_2$

Total variables ↗ ↖ dependent

$$4C_2 = \frac{4 \times 3}{1 \times 2} = 6 \text{ parameters}$$

They are $Z, h, Y, ABCD, g$ & $A'B'C'D'$

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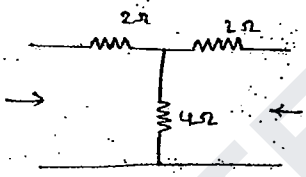
→① The N/w inside the ports is consider as Block box. where it shoul consist of Linear bilatral and passive Elements only.

→ ② They may also consist of dependent Source but never independent Sources. ③ Then they may also consist Energy Storage Elements (L & C but their initial condition assumed to be Zero.

Symmetrical Networks:-

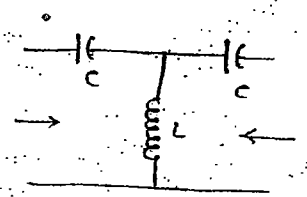
if the N/w is identical & mirror image w.r.to the two ports then it is Symmetrical.

Ex:



Symmetrical.

Ex: 2



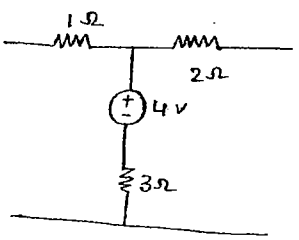
Symmetrical

Reciprocal Networks:

if the N/w obeys Reciprocity principle ie the N/w consist of only linear passive Elements ie Reciprocal.

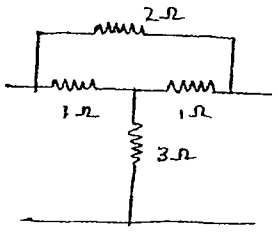
Reciprocity ⟺ passivity

Ex:



↳ non Reciprocal

Ex:



↳ Reciprocal & Symmetrical

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3] Z-parameter / O.C.I.P

↳ Here $V_1, V_2 \longrightarrow$ dependent

$I_1, I_2 \longrightarrow$ independent

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

where $Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \rightarrow$ open ckt i/p impedance (Ω)

$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \rightarrow$ open ckt Reverse transfer impedance

$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \rightarrow$ open ckt

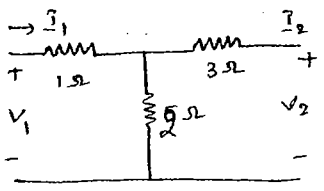
$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \rightarrow$ open ckt

Condition for Symmetrical:

$$\boxed{Z_{11} = Z_{22}}$$

Condition for Reciprocity $\boxed{Z_{12} = Z_{21}}$

Ex:



$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

$V_1 = I_1 + 6I_1$

$\frac{V_1}{I_1} = \frac{6}{2}$ $\quad \frac{V_1}{I_1} = 3\Omega$

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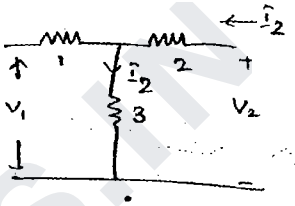
$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \qquad V_2 = 2I_1$$

$$\boxed{Z_{21} = 2\,\Omega}$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \qquad \text{KVL 2nd} \quad V_2 = 5I_2$$

$$\boxed{\frac{V_2}{I_2} = 5\,\Omega}$$

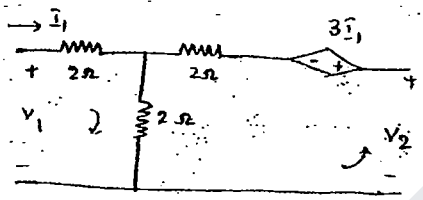


$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \qquad V_1 = 2I_2$$

$$\boxed{Z_{12} = 2\Omega}$$

Determine Z matrix:

(9)



$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad -V_1 + 2I_1 + 2I_1 = 0$$

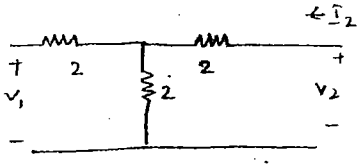
$$V_1 = 4I_1$$

$$Z_{11} = 4\Omega$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \qquad -V_2 + 3I_1 + 2I_1 = 0$$

$$V_2 = 5I_1 \Rightarrow \frac{V_2}{I_1} = 5\Omega$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$



$$Z_{22} = 4\Omega$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \qquad V_1 = 2I_2$$

$$Z_{12} = 2\Omega$$

$$Z = \begin{bmatrix} 4 & 2 \\ 5 & 4 \end{bmatrix}$$

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Y- parameters or Short ckt admittance parameters

I_1, I_2 are dependent

V_1, V_2 are independent

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$ → Short ckt i/p admittance (℧)

$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$ → Short ckt Reverse Transfer Admittance

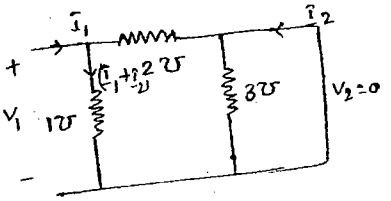
$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$ → Short ckt forward Transfer admittance

$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$ → Short ckt o/p admittance

Condition for Symetry $\boxed{Y_{11} = Y_{22}}$

Condition for Reciprocity $\boxed{Y_{12} = Y_{21}}$

(P)

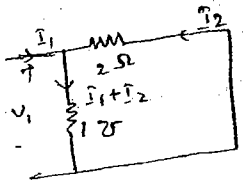


$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$

$V_1 = \frac{I_1 + I_2}{1}$ —①

3℧ is short

I_2 flows 2℧ directly



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$2\bar{I}_1 + 3\bar{I}_2 = 0$ —

$V_1 = \bar{I}_1 + \left[\frac{-2}{3}\right]\bar{I}_1$

$V_1 = \bar{I}_1\left[1 - 2/3\right] = \frac{1}{3}\bar{I}_1$

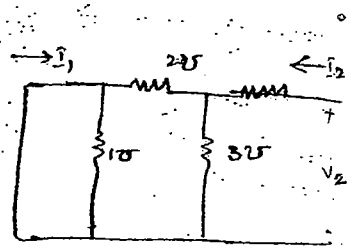
$Y_{11} = 3\mho$

$Y_{21} = \left.\frac{\bar{I}_2}{V_1}\right|_{V_2=0}$

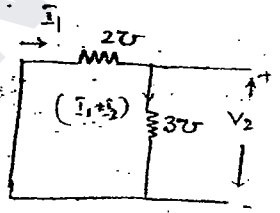
$V_1 = \frac{-3}{2}\bar{I}_2 + \bar{I}_2$

$V_1 = \frac{-1}{2}\bar{I}_2$

$Y_{21} = -2\mho$



⇒



$\frac{\bar{I}_1}{2} + \frac{\bar{I}_1+\bar{I}_2}{3} = 0$

$3\bar{I}_1 + 2\bar{I}_1 + 2\bar{I}_2 = 0$ $\quad 5\bar{I}_1 + 2\bar{I}_2 = 0$ —①

$V_2 = \frac{\bar{I}_1+\bar{I}_2}{3}$ —②

$Y_{22} = \left.\frac{\bar{I}_2}{V_2}\right|_{V_1=0}$

$V_2 = \frac{1}{3}\left[\frac{-2}{5} + 1\right]\bar{I}_2$

$\frac{V_2}{\bar{I}_2} = 1/5$ $\quad Y_{22} = 5\mho$

$Y_{12} = \left.\frac{\bar{I}_1}{V_2}\right|_{V_1=0}$

$V_2 = \frac{1}{3}\left[\bar{I}_1 - \frac{5}{2}\bar{I}_1\right]$

$= \frac{1}{3}\bar{I}_1\left[\frac{-3}{2}\right]$

$= \frac{-1}{2}\bar{I}_1$

$\frac{V_2}{\bar{I}_1} = -2\mho$

$\begin{bmatrix} 3\mho & -2\mho \\ -2\mho & 5\mho \end{bmatrix}$

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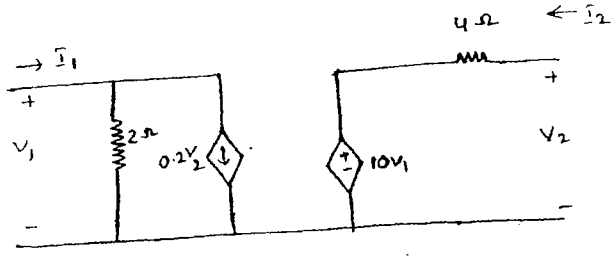
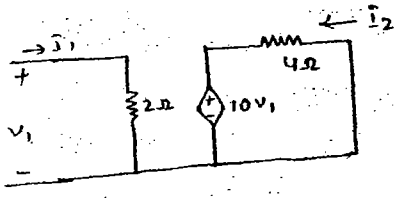
(P)



find Y matrix.

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$$



$V_2 = 0$ $\quad 0.2V_2 = 0$

open ckt

$-V_1 + 2I_1 = 0$

$V_1 = 2I_1 \qquad \frac{I_1}{V_1} = 1/2 = 0.5\,\Omega\,\mho$

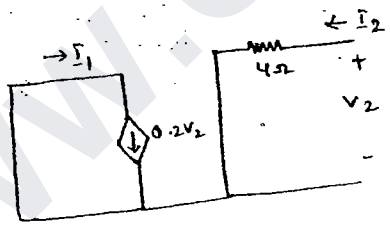
$$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$$

$4I_2 + 10V_1 = 0$

$I_2 = -2.5V_1$

$\frac{I_2}{V_1} = -2.5\,\Omega\,\mho$

$$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$



$V_2 = 4I_2$

$\frac{I_2}{V_2} = \frac{1}{4} = 0.25\,\mho$

$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$

$I_1 = 0.2V_2$

$\frac{I_1}{V_2} = 0.2\,\mho$

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hybrid ckt parameters:

↳ Here $V_1, I_2 \rightarrow$ dependent

$I_1, V_2 \rightarrow$ independent

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$ Short ckt i/p impedance (Ω)

$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$ open ckt Reverse voltage gain (unitless)

$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$ Short ckt forward Current gain (unitless)

$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$ open ckt o/p impedance (Ω)

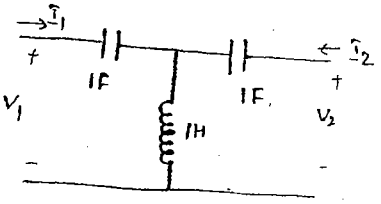
Condition for Symmetry:

$$\boxed{h_{11} h_{22} - h_{12} h_{21} = 1}$$

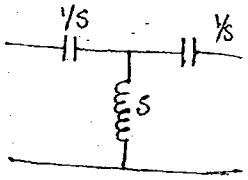
condition for Reciprocity

$$\boxed{h_{12} = -h_{21}}$$

Find h-parameter for ckt



$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$



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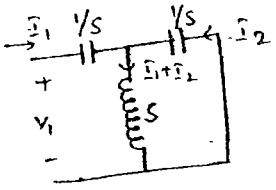
$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$



$$V_1 = \frac{I_1}{S} + (I_1+I_2)S \quad \text{—①}$$

$$\frac{I_2}{S} + (I_1+I_2)S = 0$$

$$SI_1 = -\left[\frac{1}{S}+S\right]I_2$$

$$SI_2 = -\left[\frac{S^2+1}{S}\right]I_2 \quad \text{—②}$$

$$\therefore V_1 = \frac{I_1}{S} + SI_2 + -\left[\frac{S^2+1}{S}\right]$$

$$V_1 = I_1\left[\frac{1+S^2}{S}\right] + S\left[\frac{-S^2}{S^2+1}\right]I_1$$

$$V_1 = I_1\left[\frac{1+S^2}{S} - \frac{S^3}{S^2+1}\right]$$

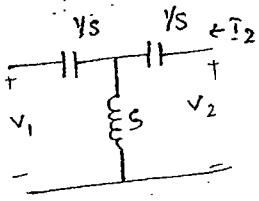
$$Y_{11} = \frac{V_1}{I_1} = \left[\frac{2S^2+1}{S(S^2+1)}\right] \mho \ \Omega$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

from ② -

$$\frac{I_2}{I_1} = \frac{-S^2}{S^2+1}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$



$$V_2 = \frac{I_2}{S} + SI_2$$

$$\frac{V_2}{I_2} = \frac{1+S^2}{S} \qquad \frac{I_2}{V_2} = h_{22} = \frac{S}{1+S^2}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \qquad V_1 = \frac{I_2}{S}S$$

$$V_1 = \frac{V_2 \cdot S}{1+S^2} \cdot S \qquad \frac{V_1}{V_2} = \frac{S^2}{1+S^2}$$

$$\begin{bmatrix} \frac{2S^2+1}{S(S^2+1)} & \frac{S^2}{S^2+1} \\ \frac{-S^2}{S^2+1} & \frac{S}{S^2+1} \end{bmatrix}$$

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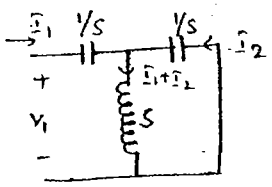
$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$



$$V_1 = \frac{I_1}{S} + (I_1+I_2)S \quad \text{—①}$$

$$\frac{I_2}{S} + (I_1+I_2)S = 0$$

$$SI_1 = -\left[\frac{1}{S}+S\right]I_2$$

$$SI_1 = -\left[\frac{S^2+1}{S}\right]I_2 \quad \text{—②}$$

$$\therefore V_1 = \frac{I_1}{S} + SI_1 + I_1\left[\frac{S^2+1}{S}\right]$$

$$V_1 = I_1\left[\frac{1+S^2}{S}\right] + S\left[\frac{-S^2}{S^2+1}\right]I_1$$

$$V_1 = I_1\left[\frac{1+S^2}{S} - \frac{S^3}{S^2+1}\right]$$

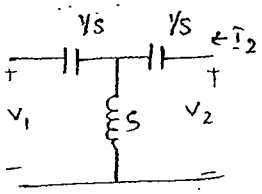
$$Y_{11} = \frac{V_1}{I_1} = \left[\frac{2S^2+1}{S(S^2+1)}\right] \mho$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

from ②

$$\frac{I_2}{I_1} = \frac{-S^2}{S^2+1}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$



$$V_2 = \frac{I_2}{S} + SI_2$$

$$\frac{V_2}{I_2} = \frac{1+S^2}{S} \qquad \frac{I_2}{V_2} = h_{22} = \frac{S}{1+S^2}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$V_1 = \frac{I_2}{S}S$$

$$V_1 = \frac{V_2 \cdot S}{1+S^2}\cdot S \qquad \frac{V_1}{V_2} = \frac{S^2}{1+S^2}$$

$$\begin{bmatrix} \frac{2S^2+1}{S(S^2+1)} & \frac{S^2}{S^2+1} \\ \frac{-S^2}{S^2+1} & \frac{S}{S^2+1} \end{bmatrix}$$

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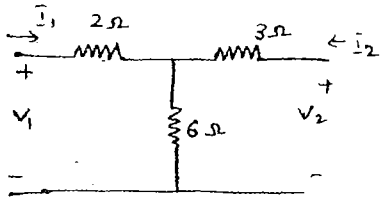
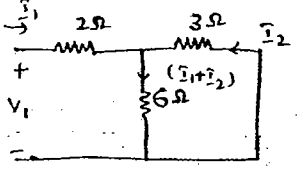
(P)



Find H-parameters

$$h_{11} = \left.\frac{V_1}{\underline{I}_1}\right|_{V_2=0}$$



$$V_1 = 2\underline{I}_1 + 6(\underline{I}_1+\underline{I}_2)$$

$$V_1 = 8\underline{I}_1 + 6\underline{I}_2$$

$$V_1 = 8\underline{I}_1 + 8\left(\frac{-6}{8}\right)\underline{I}_1$$

$$3\underline{I}_2 + (\underline{I}_1+\underline{I}_2)6 = 0$$

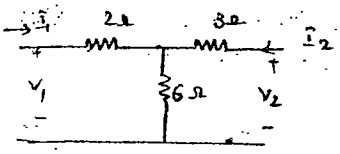
$$9\underline{I}_2 + 6\underline{I}_1 = 0$$

$$\underline{I}_2 = \frac{-6}{9}\underline{I}_1$$

$$h_{11} = \frac{V_1}{\underline{I}_1} = 4\,\Omega$$

$$h_{21} = \left.\frac{\underline{I}_2}{\underline{I}_1}\right|_{V_2=0} \qquad \underline{I}_2/\underline{I}_1 = -\frac{2}{3}$$

$$h_{22} = \left.\frac{\underline{I}_2}{V_2}\right|_{\underline{I}_1=0}$$



$$V_2 = 3\underline{I}_2 + 6\underline{I}_2$$

$$\frac{V_2}{\underline{I}_2} = 9 \Rightarrow \frac{\underline{I}_2}{V_2} = 1/9 \ \mho$$

$$h_{12} = 2/3$$

$$[h] = \begin{bmatrix} 4 & 2/3 \\ -2/3 & 1/9 \end{bmatrix}$$

WWW.GATENOTES.IN

g parameters / Inverse hybrid parameters:-

↳ Here $I_1, V_2 \rightarrow$ dependent

$V_1, I_2 \rightarrow$ independent

$$I_1 = g_{11} V_1 + g_{12} I_2$$

$$V_2 = g_{21} V_1 + g_{22} I_2$$

$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} \rightarrow$ Open ckt input admittance ($\mho$)

$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0} \rightarrow$ Short ckt Reverse Current gain

$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} \rightarrow$ open ckt forward voltage gain

$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} \rightarrow$ Short ckt O/p impedance (Ω)

Condition for symmetry:

$$[g_{11} g_{22} - g_{12} g_{21}] = 1$$

Condition for Reciprocity

$$\boxed{g_{12} = -g_{21}}$$

WWW.CATENOTES.IN

ABCD / Transmission / Chain parameters:-

↳ V_2, I_2 independent

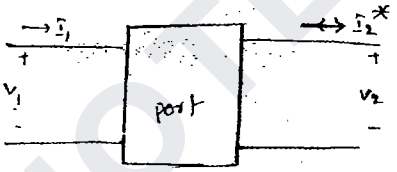
V_1, I_1 dependent

$$V_1 = AV_2 + B[-I_2]$$

$$I_1 = CV_2 + D[-I_2]$$

where

Negative Sign only indicate that the reference direction of I_2 is leaving port 2.



but parameters are not Negative. (only Current I_2 -ve)

$$A = \frac{V_1}{V_2}\Big|_{I_2=0} \longrightarrow \text{open ckt Reverse voltage gain (unitless)}$$

$$B = \frac{V_1}{-I_2}\Big|_{V_2=0} \longrightarrow \text{Short ckt Reverse Transfer impedance } (\Omega)$$

$$C = \frac{I_1}{V_2}\Big|_{I_2=0} \longrightarrow \text{open ckt Reverse Transfer admittance } (\mho)$$

$$D = \frac{I_1}{[-I_2]}\Big|_{V_2=0} \longrightarrow \text{Short ckt Reverse Current gain [unitless]}$$

Condition for Symmetry

$$A = D$$

Condition for Reciprocity

$$AD - BC = 1$$

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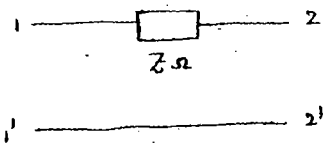
(P) Find Transmission matrix for [T] = —



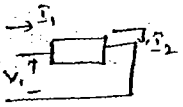
$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$I_1 = I_2$

$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ $\quad I_1 = 0$ $\quad V_1 = V_2$

$V_1/V_2 = 1$

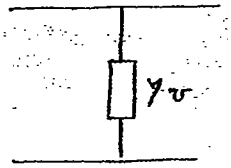
$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$



$= V_1 = 2I_1 = -2I_2$

$\frac{V_1}{(-I_2)} = \frac{2}{=}$

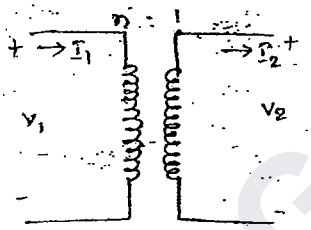
(2)



$$[T] = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

(P) Find transition matrix for Ideal T/F

(P)



$k = \frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{1}{n}$

$V_1 = nV_2 + 0\,I_2$ —①

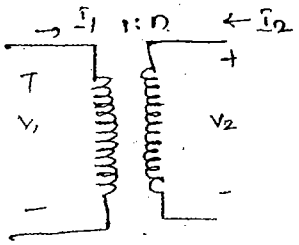
$I_1 = [0]\,V_2 + \left[\frac{1}{n}\right] I_2$

$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$

from ① $A = n$

$$[T] = \begin{bmatrix} n & 0 \\ 0 & 1/n \end{bmatrix}$$

(P)



$k = \frac{V_2}{V_1} = \frac{I_1}{-I_2} = \frac{n}{1}$

$V_1 = \left[\frac{1}{n}\right] V_2 + 0\left[I_2\right]$

$I_1 = [0]\,V_2 + \left[\frac{1}{n}\right] I_2$

$$\begin{bmatrix} 1/n & 0 \\ 0 & n \end{bmatrix}$$

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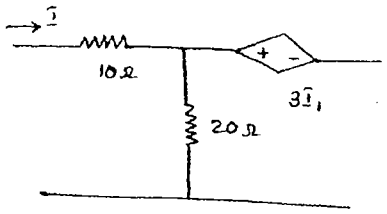
(P)



find Transmission matrix

$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ $\quad V_1 = 30\bar{I}_1 \longrightarrow (1)$

$-V_2 - 3\bar{I}_1 + 20\bar{I}_1 = 0$

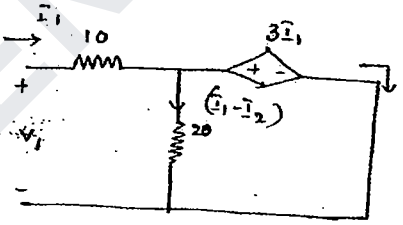
$V_2 = 17\bar{I}_1 \longrightarrow (2)$

$\frac{V_1}{V_2} = 30/17.$

$A = 30/17.$

$C = \left.\frac{\bar{I}_1}{V_2}\right|_{\bar{I}_2=0} = 1/17\ \Omega$

$\rightarrow D = \left.\frac{\bar{I}_1}{-\bar{I}_2}\right|_{V_2=0}$



$-20(\bar{I}_1 - \bar{I}_2) + 3\bar{I}_1 = 0$

$-17\bar{I}_1 + 20\bar{I}_2 = 0 \longrightarrow (1)$ $\quad \frac{\bar{I}_1}{\bar{I}_2} = \underline{\underline{20/17}}$

$D = 20/17.$

$-V_1 + 20(\bar{I}_1 - \bar{I}_2) + 10\bar{I}_1 = 0$

$30\bar{I}_1 - 20\bar{I}_2 = V_1$

$30\left(\frac{20}{17}\bar{I}_2\right) - 20\bar{I}_2 = V_1$

$\rightarrow B = \left.\frac{V_1}{-\bar{I}_2}\right|_{V_2=0}$

$B = \frac{V_1}{[-\bar{I}_2]} = \frac{260}{17}\ \Omega$

$= \frac{260}{17}\ \Omega$

$$\begin{bmatrix} \frac{30}{17} & 260/17 \\ 1/17 & 20/17 \end{bmatrix}$$

WWW.GATENOTES.IN

Inverse ABCD / Inverse Transmission parameters:-

↳ Here $V_2\ I_2$ — dependent

$V_1\ I_1$ — independent

$$V_2 = A'V_1 + B'[-I_1]$$

$$I_2 = C'V_1 + D'[-I_1]$$

Condition for Symmetry:-

$$\boxed{A' = D'}$$

Reciprocity ... $\boxed{A'D' - B'C' = 1}$

NOTE:-

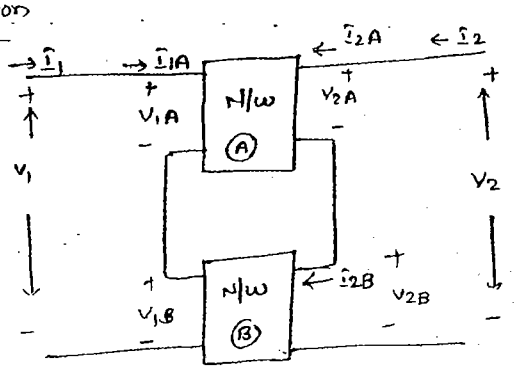
$$[Z] \longleftrightarrow [Y]^{-1}$$

$$[h] \longleftrightarrow [g]^{-1}$$

But $[T] \neq [T']^{-1}$

Interconnection of two port N/w:-

(a) Series Connection



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Here $\underline{I}_1 = \underline{I}_{1A} = \underline{I}_{1B}$

$\underline{I}_2 = \underline{I}_{2A} = \underline{I}_{2B}$

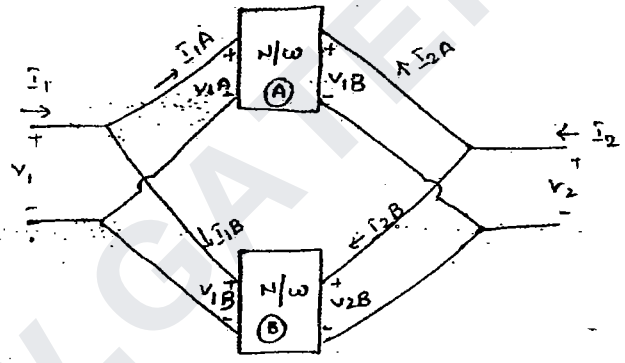
$V_1 = V_{1A} + V_{1B}$

$V_2 = V_{2A} + V_{2B}$

Represent their Z-parameter

So $\boxed{[Z_T] = [Z_A] + [Z_B]}$

(b) parallel Connection



Here $V_1 = V_{1A} = V_{1B}$ $\quad \underline{I}_1 = \underline{I}_{1A} + \underline{I}_{1B}$

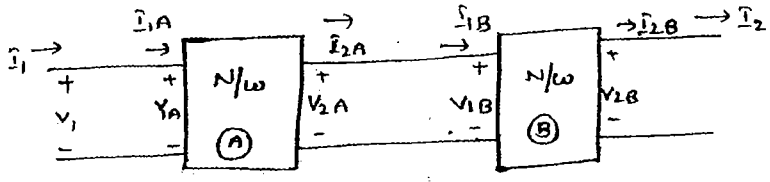
$V_2 = V_{2A} = V_{2B}$ $\quad \underline{I}_2 = \underline{I}_{2A} + \underline{I}_{2B}$

Represent their y-parameters

$$[Y_T] = [Y_1] + [Y_2]$$

WWW.GATENOTES.IN

c. Cascade Connection:



$V_1 = V_{1A}$

$V_{2A} = V_{1B}$

$V_{2B} = V_2$

$\underline{I}_1 = \underline{I}_{1A}$

$\underline{I}_{2A} = \underline{I}_{1B}$

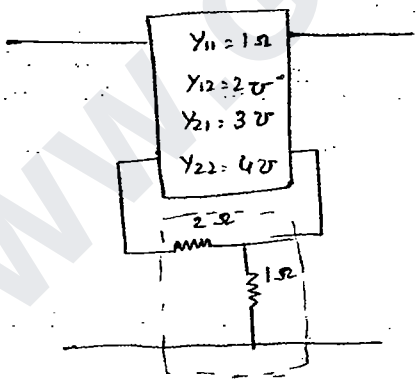
$\underline{I}_{2B} = \underline{I}_2$

Chain Connection

Representation their $[\bar{T}]$ parameter

$$[\bar{T}]_{eq} = [\bar{T}_A] \cdot [\bar{T}_B]$$

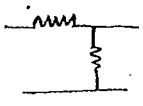
(P)



$$Z_A = [Y_A]^{-1} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^{-1}$$

$$Z_A = \frac{1}{4-6}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

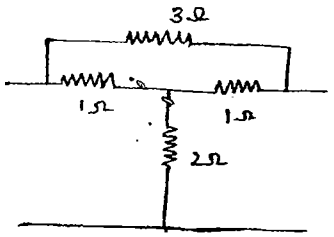
$$= \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$$

$Z_B =$  $\Rightarrow \begin{matrix} 3 & 1 \\ 1 & 1 \end{matrix}$

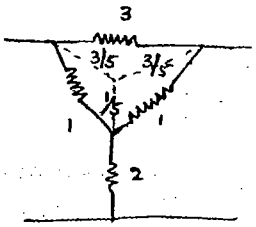
$$[Z] = Z_A + Z_B = \begin{bmatrix} 1 & 2 \\ 5/2 & 1/2 \end{bmatrix}\ \Omega$$

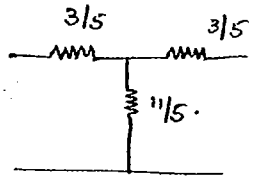
WWW.GATENOTES.IN

(P)



find [Y] = ?



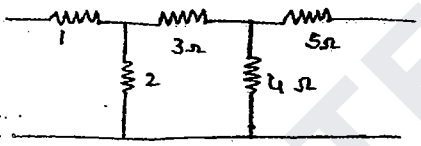


$$[Z] = \begin{bmatrix} \frac{14}{5} & \frac{11}{5} \\ \frac{11}{5} & \frac{14}{5} \end{bmatrix}$$

$$[Y] = [Z]^{-1}$$

(P)

find the Transmition matrix [T]:

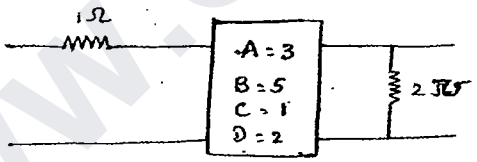


Shunt is in admitance form

$$[T] = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{1}{4} & 1 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$$

(P)

Gate



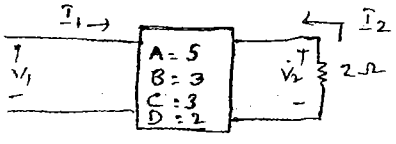
find [T] = ——

$$[T] = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

↑ for 2Ω 2℧ (Admittance)

↑ for 1Ω

(P)



find Total Zin

$V_1 = AV_2 + B[-I_2]$ $\quad V_1 = 5V_2 + 3I_2$

$V_1 = CI_2 + D[-I_2]$ $\quad I_1 = 3V_2 - (-2I_2)$

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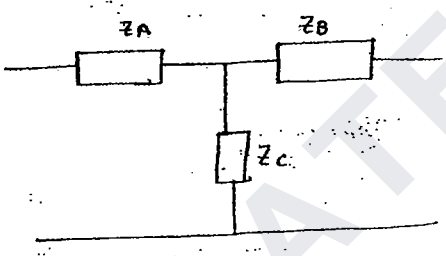
$$Z_{in} = \frac{V_1}{I_1} = \frac{5V_2 + 3I_2}{3V_2 + 2I_2}$$

But $V_2 = -2I_2$ → given

$$\frac{V_1}{I_1} = \frac{-10I_2 + 3I_2}{-6I_2 + 2I_2} = \frac{7I_2}{4I_2}$$

$$Z_{in} = \frac{V_1}{I_1} = 7/4$$

T Equivalent Representation of A 2-port Reciprocal N/w in terms of Z parameters

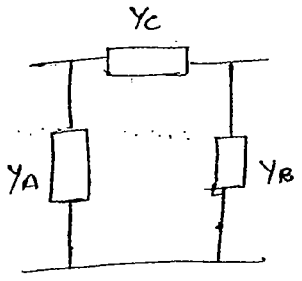


$$Z_A = Z_{11} - Z_{12}$$

$$Z_B = Z_{22} - Z_{12}$$

$$Z_C = Z_{12} = Z_{21}$$

π Equivalent of a 2port Reciprocal N/w interms of Y parameters



$$Y_A = Y_{11} + Y_{12}$$

$$Y_B = Y_{22} + Y_{12}$$

$$Y_C = Y_{12} = -Y_{21}$$

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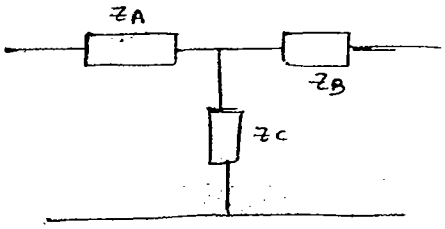
[Z] of a Reciprocal 2port N/w is $\begin{bmatrix} 2\Omega & 3\Omega \\ 3\Omega & 4\Omega \end{bmatrix}$ then the parameters of its Equivalent T model is —



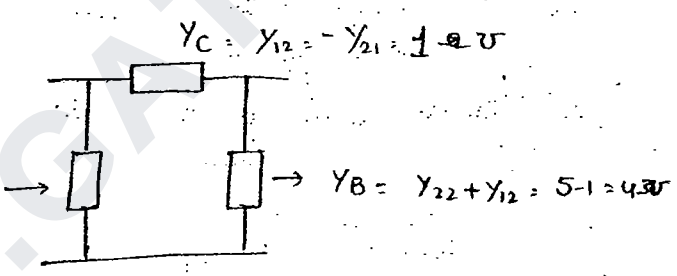
$Z_{11} - Z_{12} = Z_A = 2 - 3 = -1$

$Z_{22} - Z_{12} = Z_B = 4 - 3 = 1$

$Z_{12} = Z_{21} = Z_C = 3\Omega$

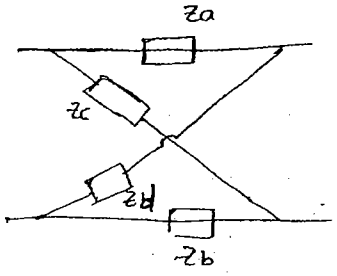
The port Currents of a 2port N/w given as $I_1 = 2.5V_1 - V_2$

$I_2 = -V_1 + 5V_2$ then its Equivalent π model is



Find Z - parameter of ckt

Lattice Network :-



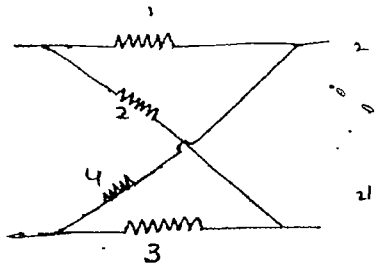
If $Z_a = Z_b$ & $Z_c = Z_d$

Symetrical

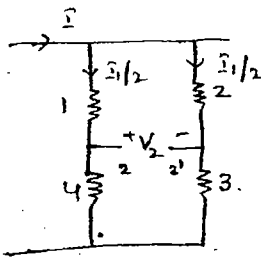
if $Z_a \neq Z_b$ & $Z_c \neq Z_d$

Un Symmetrical Lattice

WWW.GATENOTES.IN



$\Rightarrow$



$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 5\|5 = 2.5\Omega$$

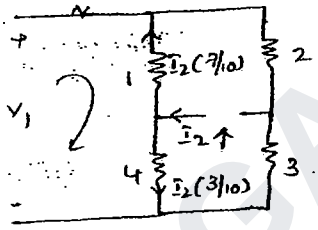
$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$-V_2 - \frac{I_1}{2} + I_1 = 0$$

$$V_2 = I_1/2$$

$$Z_{21} = 1/2\,\Omega = 0.5\Omega$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$



$$Z_{22} = 3\|7 = 2.1\,\Omega$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$-V_1 - \frac{7}{10}I_2 + \frac{6}{5}I_2 = 0$$

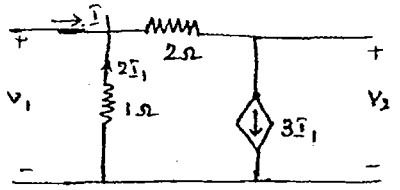
$$V_1 = \left(\frac{6}{5} - \frac{7}{10}\right)I_2 = \frac{12-7}{10} = 0.5\,I_2$$

$$Z_{12} = 0.5\Omega$$

$$[Z] = \begin{bmatrix} 2.5 & 0.5 \\ 0.5 & 2.1 \end{bmatrix}\Omega$$

WWW.GATENOTES.IN

* Find the impedance matrix.



$$Z_{11} = \left.\frac{V_1}{\bar{I}_1}\right|_{\bar{I}_2=0}$$

$V_1 = -2\bar{I}_1 \Rightarrow Z_{11} = -2\Omega$

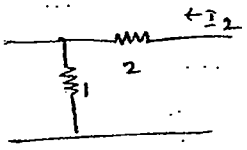
$$Z_{21} = \left.\frac{V_2}{\bar{I}_1}\right|_{\bar{I}_2=0}$$

$= 2\bar{I}_1 + 6\bar{I}_1 + V_2 = 0$

$V_2 = -8\bar{I}_1$

$\frac{V_2}{\bar{I}_1} = -8$

$$Z_{22} = \left.\frac{V_2}{\bar{I}_2}\right|_{\bar{I}_1=0}$$



$Z_{22} = 3\Omega$

-2 | -8 3

$$Z_{12} = \left.\frac{V_1}{\bar{I}_2}\right|_{\bar{I}_1=0}$$

$V_1 = \bar{I}_2$

$Z_{12} = 1\Omega$

WWW.GATENOTES.IN

Network Concepts and Filter ckts

if the magnitude of Signal is constant but frequency Varies then it is called frequency Response. Freq Response of a ckt is called Complete Response of a ckt.

→ The frequency is Varies in the N/w both magnitude & phase angle of parameter Represented as Transfer function

→ In N/w Functions we have 4 types of Transfer functions.

→ Voltage gain Transfer Func.

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$$

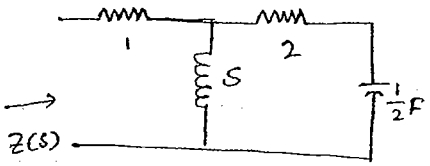
→ Current gain Transfer function

$$H(\omega) = \frac{I_o(\omega)}{I_i(\omega)}$$

→ Transfer impedance $H(\omega) = \frac{V_o(\omega)}{I_i(\omega)}$

→ Transfer admittance $H(\omega) = \frac{I_o(\omega)}{V_i(\omega)}$

(5) Find the impedance of the N/w



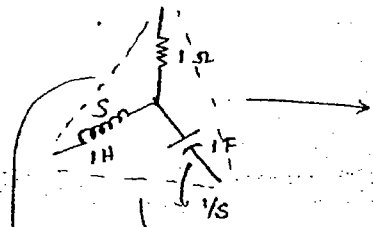
$$Z(S) = 1 + [S \,||\, (2 + \tfrac{2}{S})]$$

$$= 1 + [S \,||\, \tfrac{2S+2}{S}] = 1 + \frac{2S+2}{S^2 + 2S + 2}$$

WWW.GATENOTES.IN

$$Z(s) = \frac{2s^2+2s+2}{s^2+2s+2}\ \Omega$$

(P)



$1+\frac{1}{s}+\frac{1/s}{s} = \frac{s^2+s+1}{s^2}$

$s+\frac{1}{s}+\frac{1}{1} \Rightarrow \frac{s^2+s+1}{s}\ \Omega$

$s^2+s+1\ \Omega$

(P) Initial value of Current is $I(s) = \frac{s+4}{(s+2)(s+3)}$

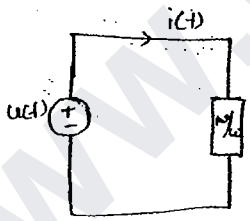
$$I(0) = \lim_{s\to\infty} s\cdot I(s)$$

$$= \lim_{s\to\infty} \frac{s\cdot(s+4)}{(s+2)(s+3)} = \lim_{s\to\infty} \frac{(1+\frac{4}{s})}{(1+\frac{2}{s})(1+\frac{3}{s})}$$

$$= \frac{1}{1} = 1A$$

ECE

(P) Gate 09



if $i(t) = e^{-3t}$ $t \geq 0$ Determine the N/w element

$H(\omega) = \frac{O/P\ (I(\omega))}{V_i}$: Yadmittance

The N/w Consist : $I_0 = e^{-3t}$ $I_0(s) = \frac{1}{s+3}$

Resistance of 1Ω & : $V_i = u(t)$ $V_i(s) = 1/s$

Capacitance of 1/3 F

$$= \frac{1/s+3}{1/s} : S/s+3 = Y(s)$$

$$Z(s) = \frac{s+3}{s} : 1+3/s = 1+\frac{1}{s[\frac{1}{3}]}$$

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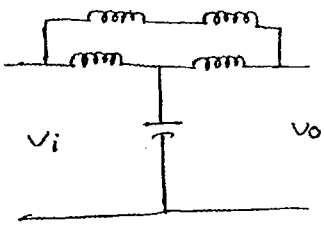
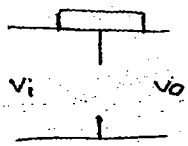
The Ckt shown below represents



$H(\omega) = \frac{V_o}{V_i}$

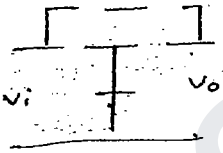
If $S \to 0$



$V_o = V_i$

$H(\omega) = 1$

If $S \to \infty$

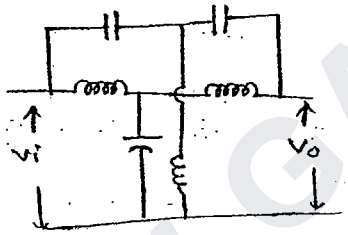


$V_i \neq V_o$

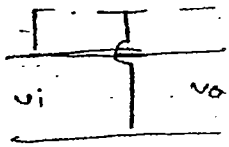
no transmission

ie Low pass filter

(P)

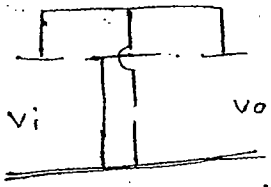


$S \to 0$

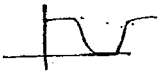


$V_i = V_o$ → Transmission

$S \to \infty$



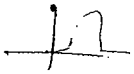
$V_o = V_i$ → Transmission



Band stop pass filter

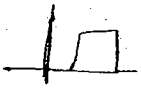
(P)

$H(\omega) = \frac{S}{S^2+S+1}$



$S=0 \to 0$

$S=\infty \to 0$

} Band pass filter 

WWW.GATENOTES.IN

3-ϕ ckts

→ n-ϕ → $\frac{360}{n}$. Except for n = 2

Phase shift of ↑

→ why 3-ϕ ?? more Simplest 1ϕ more Complex.

Two possible connection of Equipment

→ Y & Δ

Phase Sequence

The order in which 3-ϕ appearance

① R → Y → B

② R → B → Y

R - Red

Y - Yellow

B - Blue

Black - neutral

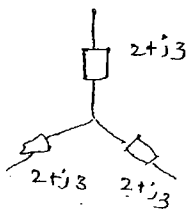
Green - Earth

Balanced Supply

→ $|V_R| = |V_Y| = |V_B|$ $\phi_{shift} = 120°$

Balance Load

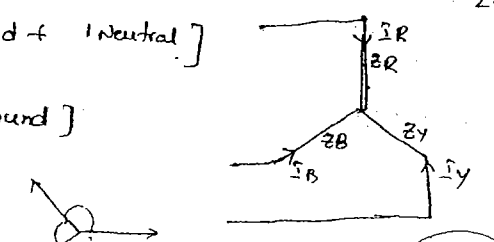
$|I_R| = |I_Y| = |I_B|$ $|\phi_R| = |\phi_Y| = |\phi_B|$

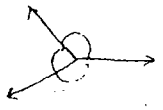


Balanced

Y → 4 wire [3 Ground + 1 Neutral]

→ 3 wire [3 Ground]





$Z_R = Z_Y = Z_B$ $I_N = 0$

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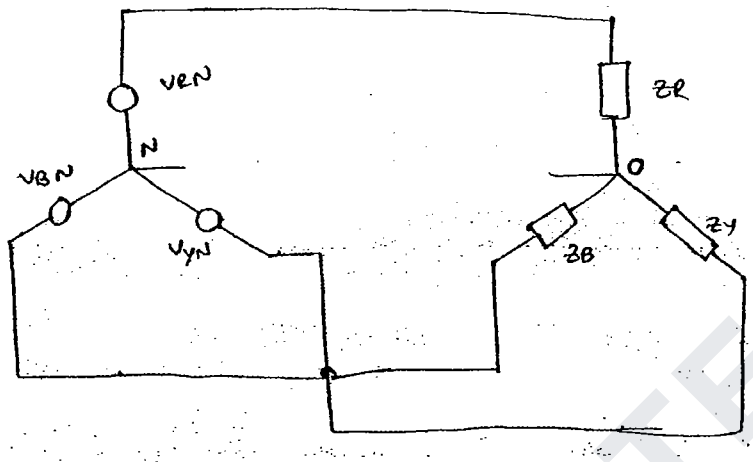
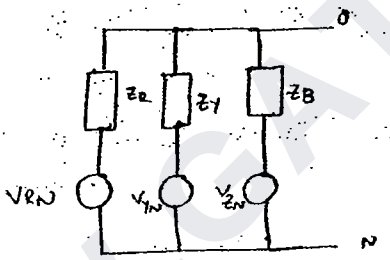
Un Balanced → we use Neutral



Un Balanced Load without Neutral.

Voltage Exist at End pa North & O point

N & O



$$V_{ON} = \frac{\frac{V_{RN}}{Z_R} + \frac{V_{YN}}{Z_Y} + \frac{V_{BN}}{Z_B}}{\frac{1}{Z_R} + \frac{1}{Z_Y} + \frac{1}{Z_B}}$$

$$V_{ON} = \frac{V_{RN} Y_R + V_{YN} Y_Y + Y_B V_{BN}}{Y_R + X_Y + Y_B}$$

Un balanced Load with Neutral

So, $V_{ON} = 0$

└ But I_N Exist

$$I_N = -[I_R + I_Y + I_B]$$

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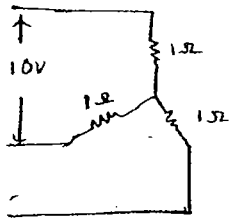
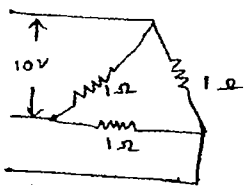
Case i



$P_Y = 3V_p I_{ph} = 3\left[\frac{10}{\sqrt{3}}\right]\left[\frac{10}{\sqrt{3}}\right]$

$= 100$

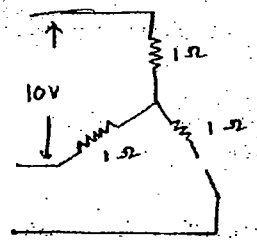
Case (b)



$3V_p I_{ph} = 3(10)(10)$

$= 300\,\omega$

(c)



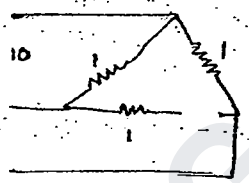
$P = \frac{(10/2)^2}{}$

$= 50$ watts

→ Two winding open power = 0

→

→



$P_\Delta = \left(\frac{10}{1}\right)^2 + \left(\frac{10}{1}\right)^2$

$= 200\,\omega$

i.e. 2/3 watts appear o/p power

Δ — $V_L = V_{ph}$

$I_L = \sqrt{3}\, I_{ph}$

$S_\Delta = 3V_L I_L$

$\frac{}{\sqrt{3}} = \sqrt{3}\, V_L I_L$

$=$

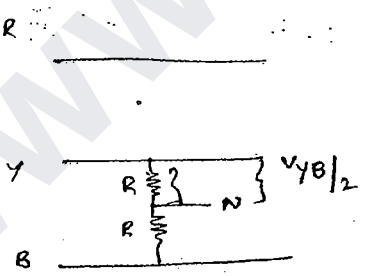
Y $V_L = \sqrt{3}\, V_{ph}$

$I_L = I_{ph}$

$S_Y = 3\frac{V_L}{\sqrt{3}} I_L$

$= \sqrt{3}\, V_L I_L$

(d)



$V_R = 100\angle 0^\circ$

$V_{RN} =$

$V_Y = 100\angle +120^\circ$

$V_{YB} = V_{YN} - V_{BN}$

$= 100\angle -120 - 100\angle -240$

$V_{YB}/_2$?

$-[V_{YB}] - V_{RY} + V_{RN} = 0$

$\frac{V_{YB}}{2} = V.$

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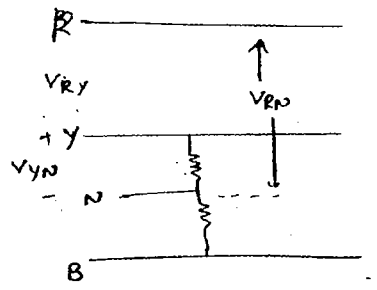

$V_{RY} = 100\angle 0 - 100\angle -120^\circ$

$= 173.2\angle 30^\circ$

$V_{YN} = \frac{V_{YB}}{2}, \quad \frac{100\angle -120 - 100\angle 240}{2}$

$86.6\angle -90^\circ$

$-V_{RY} - V_{YN} + V_{RN} = 0$

$V_{RN} = V_{RY} + V_{YN}$

$= 150\angle 0^\circ$

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Order of Ckt: The No of Energy Storage elements

Ex: R-L, R-C → 1st order

R-L-C, L-C, LLR, CCR → 2nd order

R - independent frequency

$Z_R = R$

L. $Z_L = +j\omega L$ $\boxed{s = j\omega}$ ← complex frequency

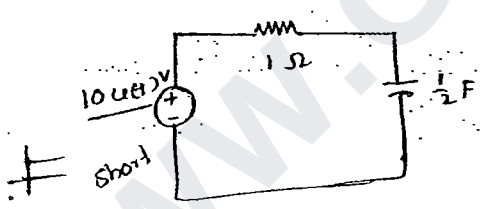
DC $Z_L = 0$

$Z_L = sL$

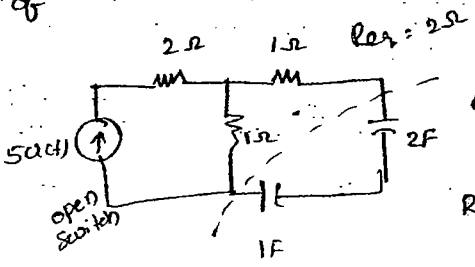
C. $Z_C = \frac{1}{j\omega C} = Z_C = \frac{1}{Cs}$

2nd order time Constant does not exist

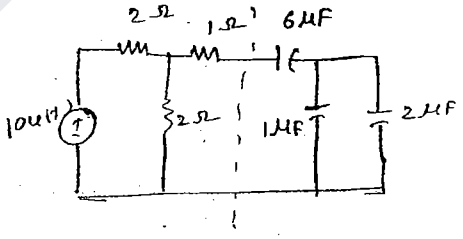
(P) Find the time Constant of



$R_C = 1 \times \frac{1}{2} = \frac{1}{2}$ Sec



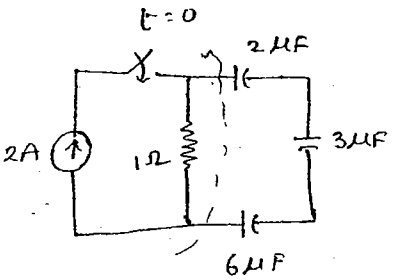
$\tau = R_{eq} C_{eq} = 2 \cdot 2/3 = 4/3$



$C_{eq} = 2\mu F$

$R_{eq} = 2\Omega$

$R_{eq} C_{eq} = 4$ Sec.

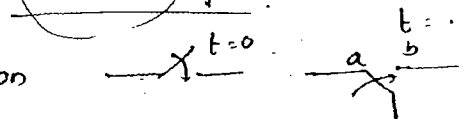


$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

$C_{eq} = 1\mu F$

$\tau = RC = 1$ s

How to identify a transient problem

→ Switch operation 

→ Any question Related to time

Find $V_R(t=1)$, E_L at t=0.1 sec

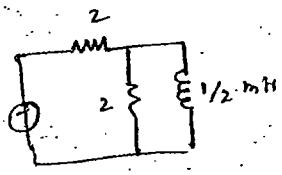
→ Initial condition problems

$i(0^+)$, $V(0^-)$, $\frac{d^2i(0^+)}{dt^2}$

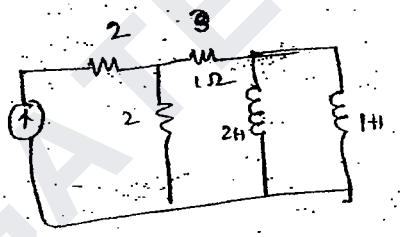
→ Special Function.

Step i/p u(t)

Ramp i/p r(t=2)

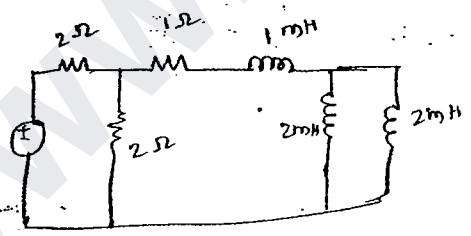


$\tau = 1 \times \frac{1}{2} = \frac{1}{2}$ sec



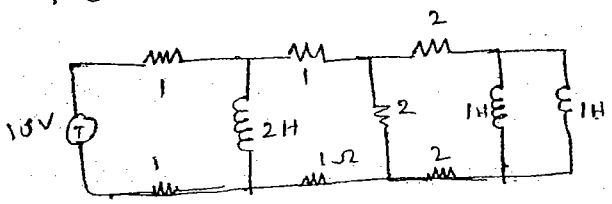
2/3 H

$\tau = \frac{2/3}{3} =$



$\tau = \frac{2}{2m_H} = 1$ msec

(P) Find 'τ' of ckt below



$\tau = \frac{L}{R}$

L_{eq} =

Here the inductors & resistors can't be lumbed or grouped at one point. Such ckt will have multiple time

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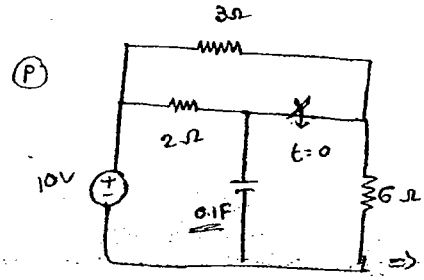
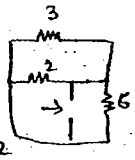
constants ie at different Sections & Solution to the item is possible by Laplace Transform method.

(P)

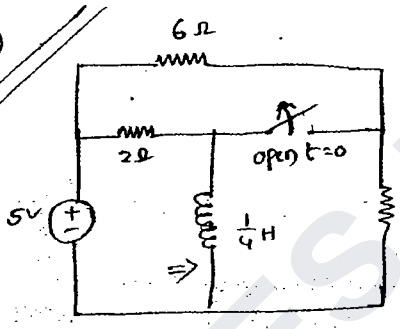




$R_{eq} = 3 \| 2 \| 6 = 1\Omega$

$\tau = 1 \times 0.1 = 0.1$ sec ✓

(P)

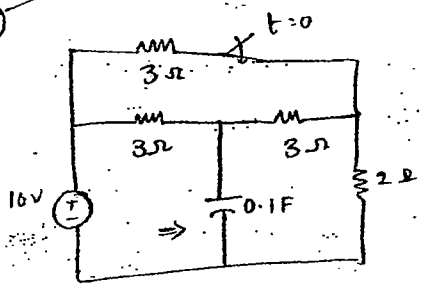


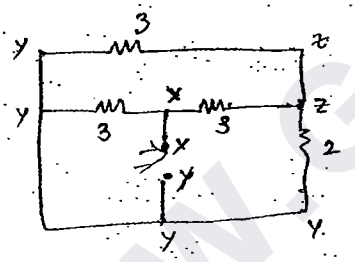
$R_{eq} = 2\Omega$

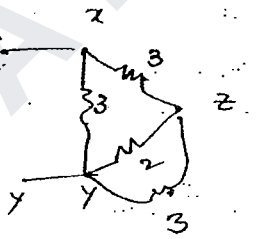
$L = 1/4$

$\tau = \frac{1/4}{1} = 1/4$ sec $T = \frac{1/4}{2} = \frac{1}{8}$

(3) IES







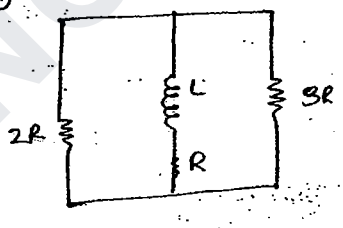
$R_{eq} = 3 \| [3+(2\|3)]$

$= 7/4$

$\tau = R_{eq} C$

$= \frac{0.7}{4}$ sec

(4)



$R_{eq} = R + [2R \| 3R]$

$= R + \left[\frac{6R^2}{5R}\right]$

$= R + \frac{6R}{5} = \frac{11R}{5}$

$\tau = \frac{L}{R} = \frac{L}{\frac{11R}{5}} = \frac{5L}{11R}$ sec

Gate

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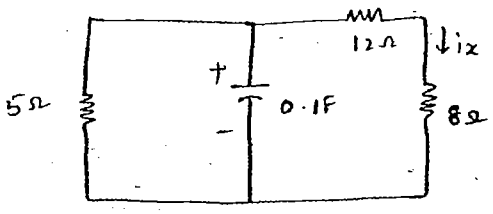
if $V(0) = 15\ V$ Find the Complete Expression for i_x.

(P)



Source Free ckt asking Response.

So Transient

Rc ckt $\quad V = V_0 e^{-t/\tau}$

$V = 15 e^{-t/0.4}$ Volts

$= 15 e^{-2.5t}$

$\tau = Req \cdot C$

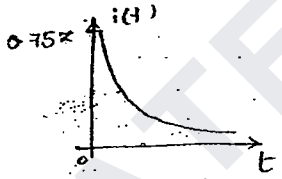
$= [20 \| 5]\ (0.1)$

$= 0.4$ sec

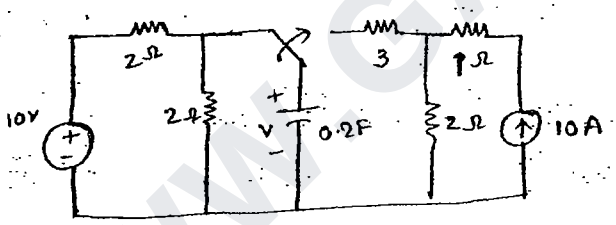
i_x for $(t>0)$ $\quad i_x = \frac{V(t)}{R} = \frac{V(t)}{12+8} = \frac{15 e^{-2.5t}}{20}$

$= 0.75 e^{-2.5t}$ Amps

$t > 0$



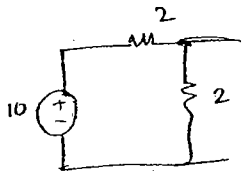
(P)



Find Complete Exp for V

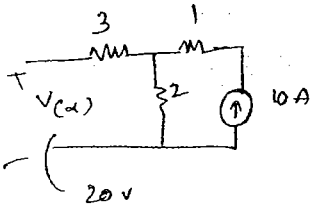
$V(t) = V(\infty) + [V(0) - V(\infty)] \cdot e^{-t/\tau}$

$V(0) =$



$V_0 = 10 \times \frac{2}{4} = 5V$

$V(\infty)$



$V(\infty) = 20V$

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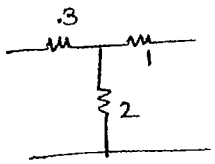
τ :



$R_{eq} = 5\ \Omega$

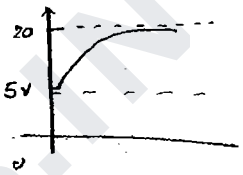
$\tau = R_{eq} \times C = 5 \times 0.2$

$= 1\ sec$

$V(t) = V(\infty) + [V(0) - V(\infty)] \cdot e^{-t/\tau}$

$= 20 + [5 - 20]\, e^{-t}$

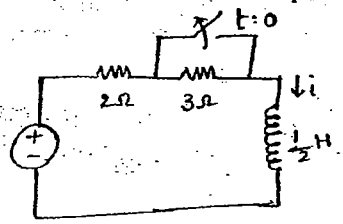
$= 20 - 15\, e^{-t}$



gate

(P)

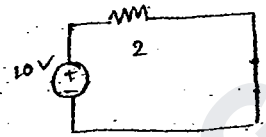
Find the Complete Exp for Current through 'L'



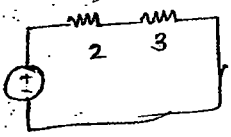
$i(t) = I(\infty) + [I(0) - I(\infty)] \cdot e^{-t/\tau}$

Coming under step Response.

Before Switch closed

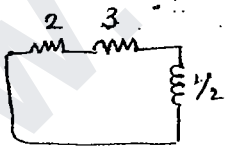


$i(0) = \frac{10}{2} = 5$



$i(\infty) = 10/5 = 2A$

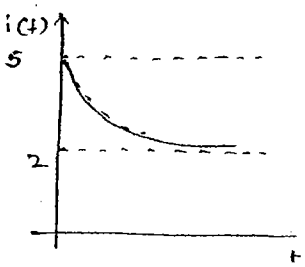
(τ)



$\tau = \frac{1/2\ (L/R)}{5} = \frac{1}{10}$

$i(t) = 2 + [5 - 2] \cdot e^{-t/1/10} = 2 + [3]\, e^{-10t}$

$= 2 + 3\, e^{-10t}$



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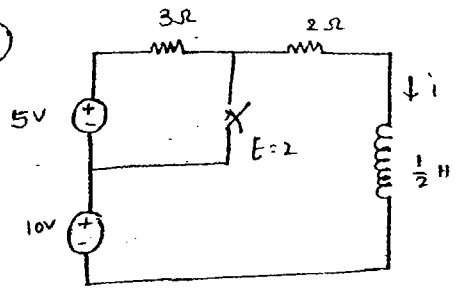
(P)

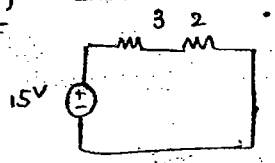


Find Complete Exp for Current through Inductor

$0<t<2$ $\quad i(t) = i(\infty) + [i(0) - i(\infty)] \cdot e^{-t/\tau}$

Normal

$i(0)$



$i(0) = \frac{15}{5} = 3A$

Complete solution is $\rightarrow$ $i(t)$ $t>0$

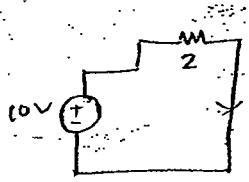
$0<t<2$

$i(\infty)$

$t \geq 2$ $\quad i(t) = i(\infty) + [I(0) - I(\infty)] e^{-t/\tau}$

But here $I(0)$ means $I(2^-)$, from possible state

$I(0) = 3A$



$\Rightarrow i(\infty) = \frac{10}{2} = 5A$

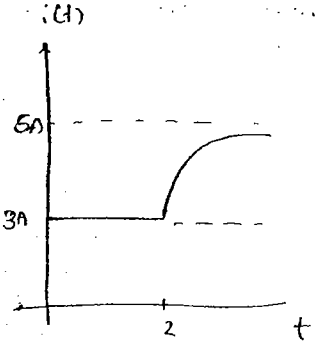
τ

$R = 2$ $\quad \tau = \frac{L}{R} = \frac{1/2}{2} = \frac{1}{4}$

$i(t) = 5 + [3-5] e^{-t/1/4} e^{-(t-2)4}$

$= 5 - 2 \cdot e^{-4(t-2)}$

$$i(t) = \begin{cases} 3A & 0<t<2 \\ 5-2\cdot e^{-4(t-2)} & t>2 \end{cases}$$



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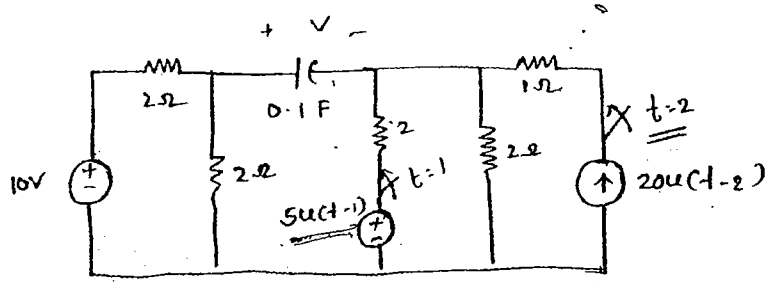
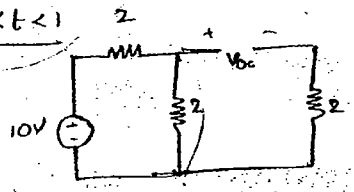
⊗ Find Complete Expression for v



$0 < t < 1$



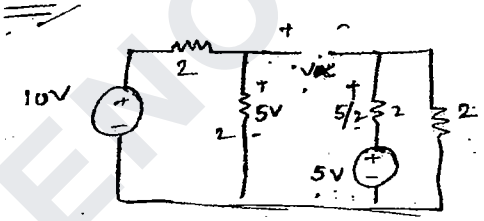
$V(0^-) = 5V$

$1 \le t \le 2$

$V(t) = V(\infty) + [V(0) - V(\infty)] \cdot e^{-t/\tau}$

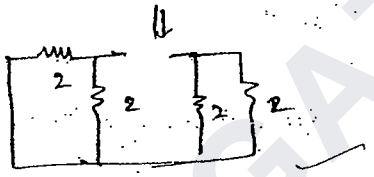
Here $V(0) \longrightarrow V(1) = 5V$

$V(\infty)$



$-5 + V(\infty) + \frac{-5}{2} + 5 = 0$

$V(\infty) = 2.5V$



$R = 2\Omega$

$\tau = RC = 2 \times 0.1$

$= 0.2$ sec

$V(t) = 2.5 + [5 - 2.5] \cdot e^{-t/0.2}$

$= 2.5 + [5 - 2.5]\, e^{-5(t-1)}$

$t > 2$

$V(t) = V(\infty) + [V(0) - V(\infty)]\, e^{-t/\tau}$

$V(0) = V(2^-)$ from previous stat

$V(t) = V(t=2) = 2.5 + [2.5]\, e^{-5(2-1)}$

$= 2.513$

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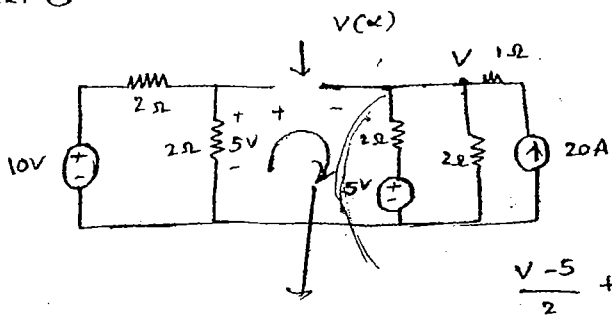
From ckt ①



$$\frac{V-5}{2} + \frac{V}{2} - 20 = 0$$

KVL

$$V = 46/2 = 23$$

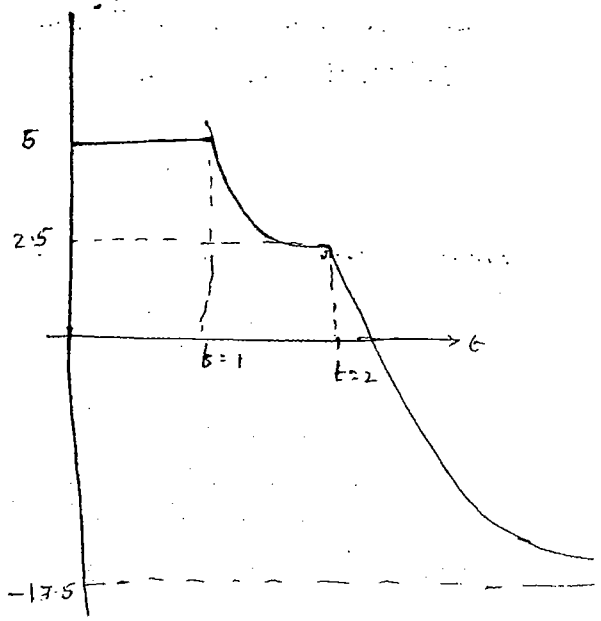
$$-5 + V(\infty) + \frac{45}{2} = 0$$

$$V(\infty) = -17.5\ V$$

$$V(t) = -17.5 + [2.513 + 17.5] \cdot e^{-5(t-2)}$$

$$= -17.5 + 20.013\, e^{-5(t-2)}$$

$$V(t) = \begin{cases} 5 & 0 < t < 1 \\ 2.5 + 2.5 \cdot e^{5(t-1)} & 1 \le t \le 2 \\ -17.5 + 20.013\, e^{-5(t-2)} & t \ge 2 \end{cases}$$



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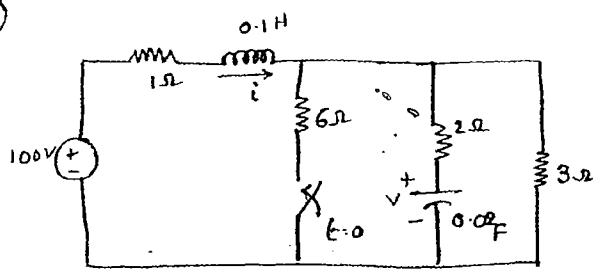
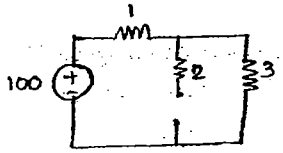
(P)



Find $i(0^+)$, $v(0^+)$, $\frac{di(0^+)}{dt}$, $\frac{dv(0^+)}{dt}$, $i(\infty)$, $v(\infty)$

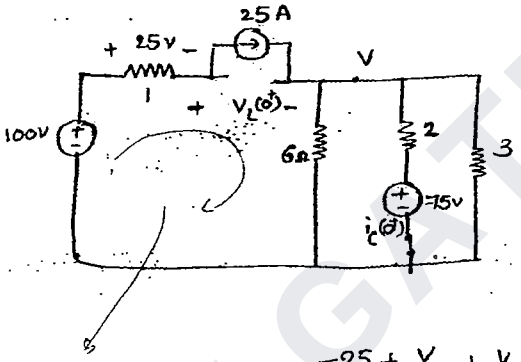
ckt for $t = 0^-$ (Flash Back)



$i(0^-) = \frac{100}{4} = 25A = i(0^+)$

$v(0^-) = 25 \times 3 = 75V = v(0^+)$

ckt at $t = 0^+$ (present)



$V_L = L\frac{di(0^+)}{dt}$

also $\begin{cases} V_L(0^+) = L\frac{di(0^+)}{dt} \\ i_c(0^+) = C\frac{dv}{dt}(0^+) \end{cases}$

KVL

$-100 + 25 + V_L(0^+) + \frac{375}{6} = 0$

$-25 + \frac{V}{6} + \frac{V-75}{2} + \frac{V}{3} = 0$

$V = \frac{375}{6}$

$V_L(0^+) = 25/2$

$\frac{di(0^+)}{dt} = \frac{V_L(0^+)}{L} = \frac{25}{2 \times 0.1} = 125 \text{ A/sec}$

Also

$i_c(0^+) = \frac{V-75}{2} = \frac{\frac{375}{6} - 75}{2} = -6.25 \text{ A}$

$\frac{dv(0^+)}{dt} = \frac{i(0^+)}{C} = \frac{-6.25}{0.02} = -312.5 \text{ V/sec}$

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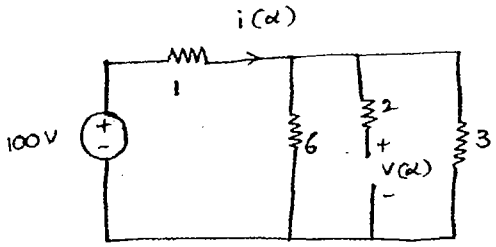
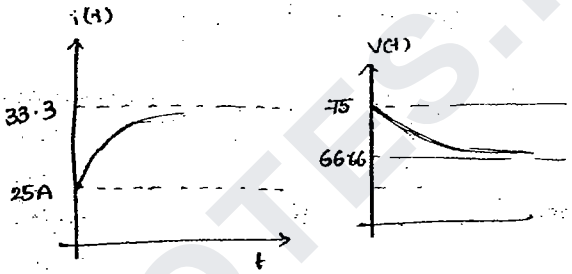
Ckt $t \to \infty$ future



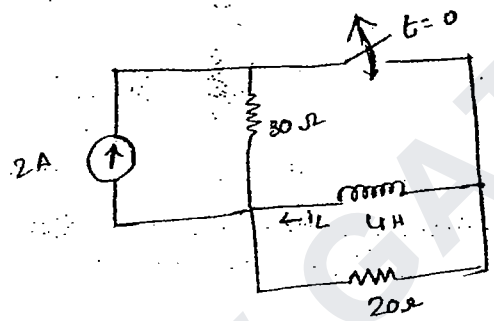
$$i(\infty) = \frac{100}{3} = 33.33\,A$$

$$V(\infty) = 3\left[\frac{100}{3} \times \frac{6}{9}\right] = 66.66V$$



(a) Find Complete Expression for i_L

(P)



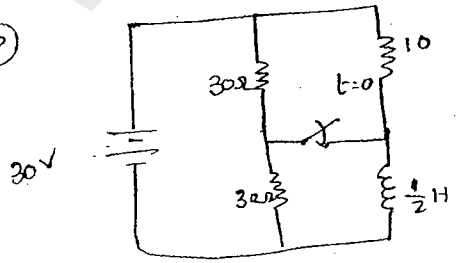
Source free

$i_L(t) = i_0\, e^{-t/\tau}$ $\quad i_0 = 2A$ because L is short

L || 20Ω Short

$\tau = \frac{L}{R} = \frac{4}{20} = 1/5$ $\quad i_L(t) = 2e^{-5t}\,A$

(P)



$$i(t) = i(\infty) + [i(0) - i(\infty)] \cdot e^{-t/\tau}$$

$6\Omega \,||\, 10 = \frac{60}{7}\Omega$

$i(0) = -3.5$

$\frac{30}{60/7} = \frac{30 \times 7}{60} = \frac{7}{2}$

$$i(\infty) = \frac{30}{10} = 3A$$

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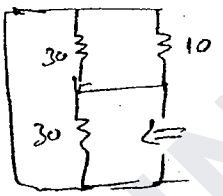
$i(\infty) = \dfrac{30}{30 \| 10} = \dfrac{30}{300} \times 40 = 4A$

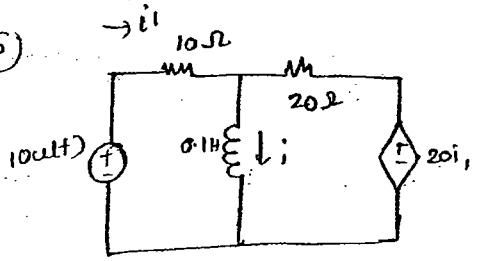
$\tau = \dfrac{L}{R} = \dfrac{1/2}{(30 \| 30 \| 30)} = \dfrac{1/2}{6} = 1/12$ Sec

$i(t) = 4 - e^{-12t}$ A



$R_{eq} = 30 \| 30 \| 10$

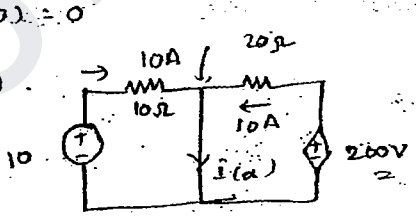
(P)



$i(t) = I(\infty) + [i(0) - I(\infty)]e^{-t/\tau}$

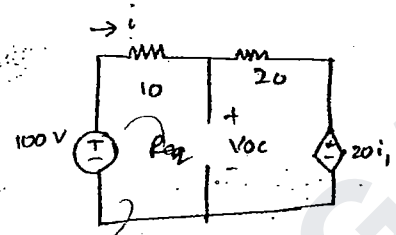
$I(0) = 0$

$I(\infty)$:



KCL : $I(\infty) = 10 + 10 = 20$

τ



Req Category II

$R_{eq} = \dfrac{V_{oc}}{I_{sc}}$

V_{oc}

$-100 + 30i_1 + 20i_1 = 0$

$i_1 = 2$

KVL

$-100 + 20 + V_{oc} = 0$

$V_{oc} = 80$

$I_{sc} = I(\infty) = 20A$

$R_{eq} = \dfrac{V_{oc}}{I_{sc}} = \dfrac{80}{20} = 4$

$\tau = \dfrac{L}{R} = \dfrac{0.1}{4} = 1/40$

$i(t) = 20\left[1 - e^{-40t}\right]$

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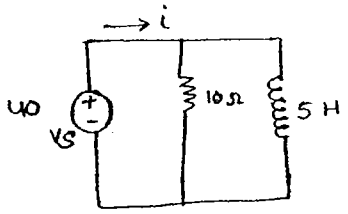
(P)

If $V_s = 40t$, & $i_L(0) = 5\,A$ then $i(t)$ at $t = 2$ sec



$i = i_R + i_L$

$$i(t) = \frac{V}{R} + \frac{1}{L}\int_{-\infty}^{t} V(s)\,ds$$

$$= \frac{40t}{10} + \left(\frac{1}{5}\int_{-\infty}^{0} V_s(t)\,dt\right) + \frac{1}{5}\int_{0}^{t} V_s(t)\,dt$$

(initial Current)

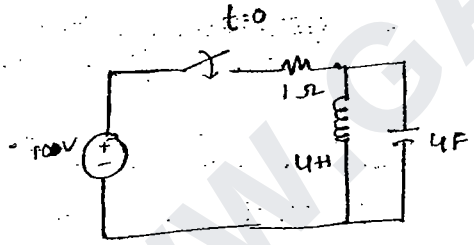
$$= 4t + 8\,i_L(0) + \frac{1}{5}\int_{0}^{t} 40t\,dt$$

$$= 4t + 5 + 8t^2$$

$$i(t) = 4t^2 + 4t + 5$$

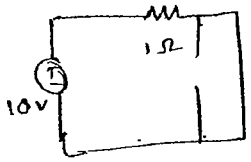
$t = 2 \Rightarrow i(t) = 29\,A$

(P)

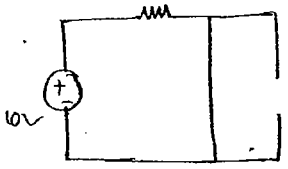


determine Current through battery at $t = 0^+$ & $t \to \infty$

$i(0^+) =$

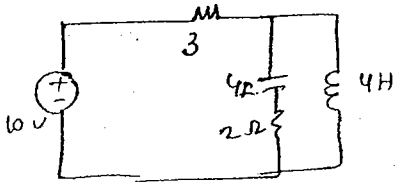


10



$i(\infty) = \frac{10}{1} = 10A$

(P)



$i(0^+) = 2A = \frac{10}{3+2}$

$i(\infty) = 10/3\,A$

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